Tema 5: Cálculo en campos escalares

Experimentación Activa

DERIVACION DE FUNCIONES IMPLICITAS

Sea una función implícita definida de la siguiente manera
F(x,y,z)=0 , en donde z(x,y)
Entonces
F(x,y, z(x,y)) = 0
Si derivamos miembro a miembro, y aplicando regla de la cadena nos queda:

$$\frac{\partial F}{\partial x}\frac{dx}{dx}+\frac{\partial F}{\partial y}\frac{dy}{dx}+\frac{\partial F}{\partial z}\frac{\boldsymbol{\partial z}}{\boldsymbol{\partial x}}=0$$

$$\frac{\partial F}{\partial x}\cdot 1+\frac{\partial F}{\partial y}\cdot 0+\frac{\partial F}{\partial z}\cdot\frac{\boldsymbol{\partial z}}{\boldsymbol{\partial x}}=0$$

$$\frac{\partial F}{\partial x}+\frac{\partial F}{\partial z}\frac{\boldsymbol{\partial z}}{\boldsymbol{\partial x}}=0$$

$$\frac{\boldsymbol{\partial z}}{\boldsymbol{\partial x}}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$$

. Hallar $\frac{dy}{dx}$ y $\frac{d^2y}{dx^2}$ para la siguiente función definida en forma implícita

$$f(x,y) = (x^2+y^2)^3 - 3(x^2+y^2) + 1 = 0$$

Donde la variable independiente es **x** y la variable dependientes **y**

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = \frac{3(x^2+y^2)^2\,2x - 6x}{3(x^2+y^2)^2\,2y - 6y} = -\frac{6x[(x^2+y^2)^2-1]}{6y[(x^2+y^2)^2-1]}$$

$$\boldsymbol{\frac{dy}{dx}} = -\boldsymbol{\frac{x}{y}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[-\frac{x}{y}\right] = -\frac{\frac{dx}{dx}y - x\frac{dy}{dx}}{y^2} = -\left[\frac{y - x\left(-\frac{x}{y}\right)}{y^2}\right]$$

$$= -\boldsymbol{\frac{x^2+y^2}{y^3}}$$

. Hallar $\frac{\partial z}{\partial x}$ y $\frac{\partial z}{\partial y}$

a) $xcosy + ycosz + zcosx = 1$

$f = xcosy + ycosz + zcosx - 1 = 0$

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}} = -\frac{\cos y - z\ senx}{-y\ sen\ z + \cos x} = \frac{z\ sen\ x - \cos y}{\cos x - y\ sen\ z}$$

$$\frac{\partial z}{\partial y} = -\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}} = -\frac{-x\ sen\ y + \cos z}{-y\ sen\ z + \cos x} = \frac{x\ sen\ y - \cos z}{\cos x - y\ sen\ z}$$

. Hallar $\frac{\partial z}{\partial x}$ y $\frac{\partial z}{\partial y}$, dado el siguiente sistema $\begin{cases} x = ucosv \\ y = usenv \\ z = cv \end{cases}$

$$\begin{cases} f_1 = x - ucosv = 0 \\ f_2 = y - usenv = 0 \\ f_3 = z - cv \ = 0 \end{cases}$$

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial(f_1, f_2, f_3)}{\partial(u, v, x)}}{\frac{\partial(f_1, f_2, f_3)}{\partial(u, v, z)}} = -\frac{\begin{bmatrix} f_{1u} & f_{1v} & f_{1x} \\ f_{2u} & f_{2v} & f_{2x} \\ f_{3u} & f_{3v} & f_{3x} \end{bmatrix}}{\begin{bmatrix} f_{1u} & f_{1v} & f_{1z} \\ f_{2u} & f_{2v} & f_{2z} \\ f_{3u} & f_{3v} & f_{3z} \end{bmatrix}} = -\frac{\begin{bmatrix} -cosv & usenv & 1 \\ -senv & -ucosv & 0 \\ 0 & -c & 0 \end{bmatrix}}{\begin{bmatrix} -cosv & usenv & 0 \\ -senv & -ucosv & 0 \\ 0 & -c & 1 \end{bmatrix}}$$

$$= -\frac{c\ sen\ v}{u\ cos^2 v + u\ sen^2 v} = -\frac{\boldsymbol{c\ sen\ v}}{\boldsymbol{u}}$$

$$\frac{\partial z}{\partial y} = -\frac{\frac{\partial(f_1, f_2, f_3)}{\partial(u, v, y)}}{\frac{\partial(f_1, f_2, f_3)}{\partial(u, v, z)}} = -\frac{\begin{bmatrix} f_{1u} & f_{1v} & f_{1y} \\ f_{2u} & f_{2v} & f_{2y} \\ f_{3u} & f_{3v} & f_{3y} \end{bmatrix}}{\begin{bmatrix} f_{1u} & f_{1v} & f_{1z} \\ f_{2u} & f_{2v} & f_{2z} \\ f_{3u} & f_{3v} & f_{3z} \end{bmatrix}} = -\frac{\begin{bmatrix} -cosv & usenv & 0 \\ -senv & -ucosv & 1 \\ 0 & -c & 0 \end{bmatrix}}{\begin{bmatrix} -cosv & usenv & 0 \\ -senv & -ucosv & 0 \\ 0 & -c & 1 \end{bmatrix}}$$

$$\frac{\partial z}{\partial y} = -\frac{-c\cos v}{u} = \boldsymbol{\frac{c\cos v}{u}}$$