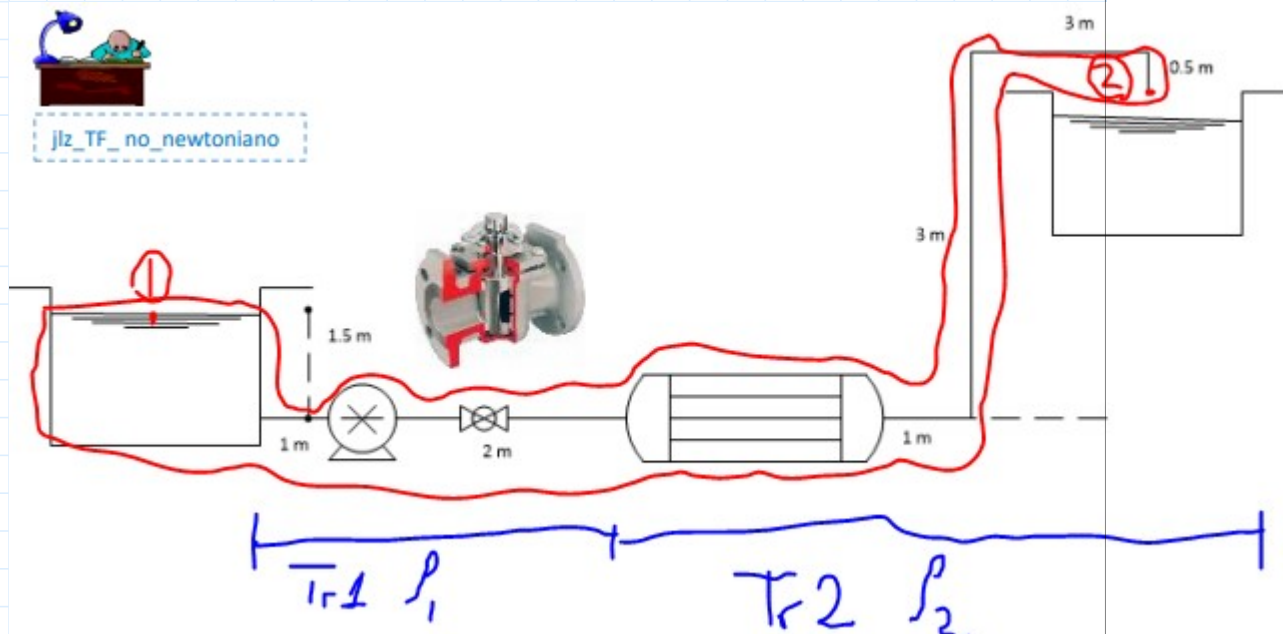


Problema y sistema

Se requiere transportar $1.97 \text{ kg}\cdot\text{s}^{-1}$ de un fluido no-Newtoniano (densidad, $1250 \text{ kg}\cdot\text{m}^{-3}$) de ambos abiertos, mediante un ducto de $1.25'' \# 40\text{s}$. La caída de presión a través del STHE incrementa de la temperatura del fluido, su densidad disminuye un 6%. Asuma que el comportamiento reológico modelable por LP. Asuma que el KL de la *plug valve* es 2. Determine la potencia necesaria de bombeo en los casos en que, a) el coeficiente de consistencia es $5.2 \text{ Pa}\cdot\text{s}^n$ y el índice reológico, 0.45 . b) el coeficiente de consistencia es $0.25 \text{ Pa}\cdot\text{s}^n$ y el índice reológico, 0.45 .



DATOS

$$\begin{aligned}
 P_1 &:= 1 \text{ atm} = (1.013 \cdot 10^5) \text{ Pa} & P_2 &:= P_1 = (1.013 \cdot 10^5) \text{ Pa} & Q_m &:= 1.97 \frac{\text{kg}}{\text{s}} & D_{nom} &:= \\
 z_1 &:= 1.5 \text{ m} & z_2 &:= 3 \text{ m} - 0.5 \text{ m} = 2.5 \text{ m} & \delta_1 &:= 1250 \frac{\text{kg}}{\text{m}^3} & \delta_2 &:= 1175 \frac{\text{kg}}{\text{m}^3} & Long_1 &:= 1 \text{ m} - \\
 \Delta P &:= 100 \text{ kPa} = (1 \cdot 10^5) \text{ Pa} & Long_2 &:= 1 \text{ m} + 3 \text{ m} + 3 \text{ m} + 0.5 \text{ m} = 7.5 \text{ m} & & & & & & \\
 v_1 &:= 0 \frac{\text{m}}{\text{s}} & v_2 &:= \frac{4 \cdot Q_m}{\pi \cdot D^2 \cdot \delta_2} = 1.737 \frac{\text{m}}{\text{s}} & Q_{v1} &:= \frac{Q_m}{\delta_1} = (1.58 \cdot 10^{-3}) \frac{\text{m}^3}{\text{s}} & Q_{v2} &:= \frac{Q_m}{\delta_2} = (1.6
 \end{aligned}$$

Incógnitas

Pot_{bomb}

Ecuaciones

$$v(Q_m, D, \delta) := \frac{4 \cdot Q_m}{\pi \cdot D^2 \cdot \delta}$$

$$GRey(K_{coef}, D, Q_m, \delta, n_{indice}) := \frac{D^{n_{indice}} \cdot v(Q_m, D, \delta)^{(2-n_{indice})} \cdot \delta \cdot \left(\frac{4 \cdot n_{indice}}{3 \cdot n_{indice} + 1} \right)^{n_{indice}}}{8^{(n_{indice}-1)} \cdot K_{coef}}$$

$$GRey_{crit}(n_{indice}) := \frac{6464 \cdot n_{indice} \cdot (2 + n_{indice})^{\frac{2+n_{indice}}{1+n_{indice}}}}{(1 + 3 \cdot n_{indice})^2}$$

$$f_D(K_{coef}, D, Q_m, \delta, n_{indice}) := \frac{64}{GRey(K_{coef}, D, Q_m, \delta, n_{indice})}$$

$$\alpha_{RL}(n_{indice}) := \frac{3 \cdot (3 \cdot n_{indice} + 1)^2}{(2 \cdot n_{indice}) \cdot (5 \cdot n_{indice} + 3)}$$

$$DP(Temp) := \left(\frac{P_1}{\delta} + g \cdot z_1 + \alpha_1 \cdot \frac{v_1^2}{2} \right) - \left(\frac{P_2}{\delta} + g \cdot z_2 + \alpha_2 \cdot \frac{v_2^2}{2} \right)$$

$$Ft_{RL}(K_{coef}, D, Q_m, \delta, n_{indice}) := \frac{500}{GRey(K_{coef}, D, Q_m, \delta, n_{indice})}$$

Irrever por trayectoria

$$e_{Lp}(K_{coef}, D, Q_m, \delta, n_{indice}, Long) := f_D(K_{coef}, D, Q_m, \delta, n_{indice}) \cdot \frac{Long}{D} \cdot \frac{\left(\frac{4 \cdot Q_m}{\pi \cdot D^2 \cdot \delta} \right)^2}{2}$$

Irrever por entrada

$$K_{Lcon} := 0.5$$

$$e_{Lcon}(K_{coef}, D, Q_m, \delta, n_{indice}) := Ft_{RL}(K_{coef}, D, Q_m, \delta, n_{indice}) \cdot K_{Lcon} \cdot \frac{\left(\frac{4 \cdot Q_m}{\pi \cdot D^2 \cdot \delta}\right)^2}{2}$$

Irrever por válvula de regulación

$$K_{Lvalpl}(K_{coef}, D, Q_m, \delta, n_{indice}) := 3 \cdot f_D(K_{coef}, D, Q_m, \delta, n_{indice})$$

$$e_{Lvalpl}(K_{coef}, D, Q_m, \delta, n_{indice}) := Ft_{RL}(K_{coef}, D, Q_m, \delta, n_{indice}) \cdot K_{Lvalpl}(K_{coef}, D, Q_m, \delta, n_{indice}) \cdot \frac{\left(\frac{4 \cdot Q_m}{\pi \cdot D^2 \cdot \delta}\right)^2}{2}$$

Irrever por equipo

$$e_{LSTHE}(\Delta P, \delta) := \frac{\Delta P}{\delta}$$

Irrever por accesorio, codos 90°

$$K_L(K_{coef}, D, Q_m, \delta, n_{indice}) := 30 \cdot f_D(K_{coef}, D, Q_m, \delta, n_{indice})$$

$$e_{Lacc}(K_{coef}, D, Q_m, \delta, n_{indice}) := Ft_{RL}(K_{coef}, D, Q_m, \delta, n_{indice}) \cdot K_L(K_{coef}, D, Q_m, \delta, n_{indice}) \cdot \frac{\left(\frac{4 \cdot Q_m}{\pi \cdot D^2 \cdot \delta}\right)^2}{2}$$

DESARROLLO

INCISO A

$$n_{indice} := 0.45$$

$$K_{coef} := 5.2 \text{ Pa} \cdot \text{s}^{0.45} = 5.2 \frac{\text{kg}}{\text{m} \cdot \text{s}^{\frac{31}{20}}}$$

$$GRey_{crit}(n_{indice}) = 2.394 \cdot 10^3$$

Tramo 1

Tramo 2

$$GRey(K_{coef}, D, Q_m, \delta_1, n_{indice}) = 316.834$$

$$GRey(K_{coef}, D, Q_m, \delta_2, n_{indice}) = 327.802$$

$$f_D(K_{coef}, D, Q_m, \delta_1, n_{indice}) = 0.202$$

$$f_D(K_{coef}, D, Q_m, \delta_2, n_{indice}) = 0.195$$

$$\alpha_{RL}(n_{indice}) = 3.506$$

$$DP := \left(\frac{P_1}{\delta_1} + g \cdot z_1 + \alpha_{RL}(n_{indice}) \cdot \frac{v_1^2}{2} \right) - \left(\frac{P_2}{\delta_2} + g \cdot z_2 + \alpha_{RL}(n_{indice}) \cdot \frac{v_2^2}{2} \right)$$

$$DP = -20.273 \frac{m^2}{s^2}$$

Cálculo de irreversibilidades

Tramo 1

$$e1 := \begin{bmatrix} e_{Lp}(K_{coef}, D, Q_m, \delta_1, n_{indice}, Long_1) \\ e_{Lvalpl}(K_{coef}, D, Q_m, \delta_1, n_{indice}) \\ e_{Lcon}(K_{coef}, D, Q_m, \delta_1, n_{indice}) \end{bmatrix} = \begin{bmatrix} 23.057 \\ 1.275 \\ 1.052 \end{bmatrix} \frac{m^2}{s^2}$$

$$e_{L1} := \sum_{j=0}^2 e1_j = 25.385 \frac{m^2}{s^2}$$

Tramo 2

$$e2 := \begin{bmatrix} e_{Lp}(K_{coef}, D, Q_m, \delta_2, n_{indice}, Long_2) \\ e_{LSTHE}(\Delta P, \delta_2) \\ 2 \cdot e_{Lacc}(K_{coef}, D, Q_m, \delta_2, n_{indice}) \end{bmatrix} = \begin{bmatrix} 63.054 \\ 85.106 \\ 26.97 \end{bmatrix} \frac{m^2}{s^2}$$

$$e_{L2} := \sum_{j=0}^2 e2_j = 175.13 \frac{m^2}{s^2}$$

Valores de prueba

$$w_s := 1 \frac{m^2}{s^2}$$

Restricciones

$$DP + w_s = e_{L2} + e_{L1}$$

Solver

$$w_s := \text{find}(w_s) = 220.788 \frac{m}{s}$$

$$Pot_{bomb} := w_s \cdot Q_m = 0.435 \text{ kW}$$

$$H_s := \frac{w_s}{g} = 22.514 \text{ m}$$

INCISO b

$$n_{indice} := 0.45$$

$$K_{coef} := 0.25 \text{ Pa} \cdot s^{0.45} = 0.25 \frac{kg}{m \cdot s^{\frac{31}{20}}}$$

$$GRey_{crit}(n_{indice}) = 2.394 \cdot 10^3$$

Tramo 1

Tramo 2

$$GRey(K_{coef}, D, Q_m, \delta_1, n_{indice}) = 6.59 \cdot 10^3$$

$$GRey(K_{coef}, D, Q_m, \delta_2, n_{indice}) = 6.818 \cdot 10^3$$

$$f_D(K_{coef}, D, Q_m, \delta_1, n_{indice}) = 0.01$$

$$f_D(K_{coef}, D, Q_m, \delta_2, n_{indice}) = 0.009$$

$$\alpha_{RT} := 1$$

$$DP := \left(\frac{P_1}{\delta_1} + g \cdot z_1 + \alpha_{RT} \cdot \frac{v_1^2}{2} \right) - \left(\frac{P_2}{\delta_2} + g \cdot z_2 + \alpha_{RT} \cdot \frac{v_2^2}{2} \right)$$

$$DP = -16.49 \frac{m^2}{s^2}$$

$$Ft_{RL}(K_{coef}, D, Q_m, \delta, n_{indice}) := 1$$

de prueba

$$f_f := 1$$

Restricciones/valores

$$\frac{1}{\sqrt[2]{f_f}} = \left(\frac{4}{n_{indice}^{0.75}} \right) \cdot \log \left(GRey (K_{coef}, D, Q_m, \delta_1, n_{indice}) \cdot f_f^{\left(1 - \frac{n_{indice}}{2}\right)} \right) - \left(\frac{0.4}{n_{indice}^{1.2}} \right)$$

Solver

$$f_{f1} := \mathbf{find} (f_f) = 5.2 \cdot 10^{-3}$$

$$f_{D1} := 4 \cdot f_{f1} = 0.02079$$

$$e_{Lp1} (K_{coef}, D, Q_m, \delta, n_{indice}, Long) := f_{D1} \cdot \frac{Long}{D} \cdot \frac{\left(\frac{4 \cdot Q_m}{\pi \cdot D^2 \cdot \delta} \right)^2}{2}$$

$$e_{Lp2} (K_{coef}, D, Q_m, \delta, n_{indice}, Long) := f_{D2} \cdot \frac{Long}{D} \cdot \frac{\left(\frac{4 \cdot Q_m}{\pi \cdot D^2 \cdot \delta} \right)^2}{2}$$

Irrever por entrada

$$K_{Lcon} := 0.5$$

$$e_{Lcon} (K_{coef}, D, Q_m, \delta, n_{indice}) := Ft_{RL} (K_{coef}, D, Q_m, \delta, n_{indice}) \cdot K_{Lcon} \cdot \frac{\left(\frac{4 \cdot Q_m}{\pi \cdot D^2 \cdot \delta} \right)^2}{2}$$

Irrever por válvula de regulación

$$K_{Lvalpl} := 3 \cdot f_{D1}$$

$$e_{Lvalpl} (K_{coef}, D, Q_m, \delta, n_{indice}) := Ft_{RL} (K_{coef}, D, Q_m, \delta, n_{indice}) \cdot K_{Lvalpl} \cdot \frac{\left(\frac{4 \cdot Q_m}{\pi \cdot D^2 \cdot \delta} \right)^2}{2}$$

Irrever por equipo

$$e_{LSTHE} (\Delta P, \delta) := \frac{\Delta P}{\delta}$$

Irrever por accesorio, codos

90°

$$K_L := 30 \cdot f_{D2}$$

$$e_{Lacc}(K_{coef}, D, Q_m, \delta, n_{indice}) := Ft_{RL}(K_{coef}, D, Q_m, \delta, n_{indice}) \cdot K_L \cdot \frac{\left(\frac{4 \cdot Q_m}{\pi \cdot D^2 \cdot \delta}\right)^2}{2}$$

Cálculo de irreversibilidades

Tramo 1

$$e1 := \begin{bmatrix} e_{Lp1}(K_{coef}, D, Q_m, \delta_1, n_{indice}, Long_1) \\ e_{Lvalpl}(K_{coef}, D, Q_m, \delta_1, n_{indice}) \\ e_{Lcon}(K_{coef}, D, Q_m, \delta_1, n_{indice}) \end{bmatrix} = \begin{bmatrix} 2.373 \\ 0.083 \\ 0.667 \end{bmatrix} \frac{m^2}{s^2}$$

$$e_{L1} := \sum_{j=0}^2 e1_j = 3.123 \frac{m^2}{s^2}$$

Tramo 2

$$e2 := \begin{bmatrix} e_{Lp2}(K_{coef}, D, Q_m, \delta_2, n_{indice}, Long_2) \\ e_{LSTHE}(\Delta P, \delta_2) \\ 2 \cdot e_{Lacc}(K_{coef}, D, Q_m, \delta_2, n_{indice}) \end{bmatrix} = \begin{bmatrix} 6.637 \\ 85.106 \\ 1.861 \end{bmatrix} \frac{m^2}{s^2}$$

$$e_{L2} := \sum_{j=0}^2 e2_j = 93.605 \frac{m^2}{s^2}$$

Valores de prueba

$$w_s := 1 \frac{m^2}{s^2}$$

Restricciones

$$DP + w_s = e_{L2} + e_{L1}$$

Solver

$$w_s := \text{find}(w_s) = 113.218 \frac{m^2}{s^2}$$

$$Pot_{bomb} := w_s \cdot Q_m = 0.223 \text{ kW}$$

$$H_s := \frac{w_s}{g} = 11.545 \text{ m}$$