TABLAS DE DERIVADAS E INTEGRALES

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PRÓLOGO

Para los alumnos de las distintas Facultades donde estudian Análisis Matemático y ante el requerimiento de los mismos, se ofrecen estas tablas de derivadas y de integrales que redundarán, sin duda, en su beneficio.
Encontrarán aquí la mayoría de los casos que se les pueden presentar (en total más de 800); no obstante, es conveniente destacar que las tablas no deben emplearse mecánicamente, sino atendiendo a la índole de los problemas que se les planteen y a los valores que estén en juego.

El autor

ÍNDICE ANALÍTICO

A. TABLA DE DERIVADAS

Dada la función $y = f(x)$, la función derivada es $\dfrac{dy}{dx} = f'(x) = F(x)$ [I] .

OBSERVACIONES

1. Para indicar la derivada de una función $y = f(x)$, suelen utilizarse cualesquiera de las siguientes notaciones:

$$\frac{dy}{dx};\ \frac{df}{dx};\ Df(x);\ \frac{d}{dx}y;\ \frac{d}{dx}f(x);\ y'$$

que se leen "derivada de y con repecto a x".
Por comodidad emplearemos y'.

2. Con a, c ó k indicamos las constantes, mientras que u, v ó w son funciones de x, que es la variable independiente.

3. Con $\log_a x$ indicamos logaritmo de base cualquiera, de x.
En particular llamamos logaritmo natural de x, que indicamos $\ln x$ ó Lx, al logaritmo de base e, es decir $\ln x = Lx = log_e x$.

4. No debe confundirse la función recíproca con la función inversa.

Si $f(x) = \cos x$, su recíproca es $r(x) = \dfrac{1}{\cos x}$

Si $f(x) = \cos x$, su inversa es $f^{-1}(x) = \arccos x$

Con respecto a las fuciones hiperbólicas inversas, si por ejemplo, $f: \mathbf{R} \to \mathbf{R} / f(x) = \operatorname{sh} x$ existe $f^{-1}: \mathbf{R} \to \mathbf{R} / f^{-1}(x) = \arg \operatorname{sh} x$; utilizaremos esta notación "arg. sh x", que se lee "argumento cuyo seno hiperbólico es x".

Veamos la tabla:

1.	$y = c$	$y' = 0$
2.	$y = x$	$y' = 1$
3.	$y = ax$	$y' = a$
4.	$y = au$	$y' = a\,u'$
5.	$y = u + v + w$	$y' = u' + v' + w'$
6.	$y = u - v$	$y' = u' - v'$
7.	$y = uv$	$y' = u'v + uv'$
8.	$y = uvw$	$y' = u'vw + uv'w + uvw'$
9.	$y = \dfrac{u}{v}$	$y' = \dfrac{u'v - uv'}{v^2}$
10.	$y = x^n$	$y' = n\,x^{n-1}$
11.	$y = x^n + k$	$y' = n\,x^{n-1}$
12.	$y = u^n$	$y' = n\,u^{n-1}u'$
13.	$y = \dfrac{x^n}{n}$	$y' = x^{n-1}$
14.	$y = \dfrac{u^n}{n}$	$y' = u^{n-1}\,u'$
15.	$y = \sqrt{x} = x^{1/2}$	$y' = \dfrac{1}{2}x^{-1/2} = \dfrac{1}{2x^{1/2}} = \dfrac{1}{2\sqrt{x}}$
16.	$y = \sqrt{u}$	$y' = \dfrac{u'}{2\sqrt{u}}$
17.	$y = \sqrt[n]{x}$	$y' = \dfrac{1}{n\ \sqrt[n]{x^{n-1}}}$
18.	$y = \sqrt[n]{u}$	$y' = \dfrac{u'}{n\sqrt[n]{u^{n-1}}} = \dfrac{\sqrt[n]{u}\ \ u'}{nu}$
19.	$y = \mathrm{L}x$	$y' = \dfrac{1}{x}$
20.	$y = \mathrm{L}u$	$y' = \dfrac{u'}{u}$
21.	$y = a^x$ (con $a > 0$ y $a \neq 1$)	$y' = a^x \cdot \mathrm{L}a$
22.	$y = a^u$ (con $a > 0$ y $a \neq 1$)	$y' = a^u \cdot \mathrm{L}a\, u'$
23.	$y = \dfrac{a^x}{\ln a}$	$y' = a^x$

24.	$y = \frac{a^u}{\ln a}$	$y' = a^u \cdot u'$
25.	$y = e^x$	$y' = e^x$
26.	$y = e^u$	$y' = e^u\, u'$
27.	$y = u^v$	$y' = v\, u^{v-1}\, u' + u^v\, L\, u\, v'$
28.	$y = \log_a x$	$y' = \log_a e \frac{1}{x}$
29.	$y = \log_a u$	$y' = \log_a e \frac{1}{u} u'$
30.	$y = \text{sen}\, x$	$y' = \cos x$
31.	$y = \text{sen}\, u$	$y' = \cos u\, u'$
32.	$y = \cos x$	$y' = -\,\text{sen}\, x$
33.	$y = \cos u$	$y' = -\,\text{sen}\, u\, u'$
34.	$y = \text{tg}\, x$	$y' = \frac{1}{\cos^2 x} = \sec^2 x = 1 + \text{tg}^2 x$
35.	$y = \text{tg}\, u$	$y' = \frac{u'}{\cos^2 u} = \sec^2 u\ \ u' = (1 + \text{tg}^2 u)\, u'$
36.	$y = \text{cotg}\, x$	$y' = -\frac{1}{\text{sen}^2 x} = -\text{cosec}\, x = -(1 + \text{cotg}^2 x)$
37.	$y = \text{cotg}\, u$	$y' = -\frac{u'}{\text{sen}^2 u} = -\text{cosec}\, u\ \ u' = -(1 + \text{cotg}^2 u)\, u'$
38.	$y = \sec x$	$y' = \frac{\text{sen}\, x}{\cos^2 x} = \text{tg}\, x\ \ \sec x = \text{sen}\, x\ \ \sec^2 x$
39.	$y = \sec u$	$y' = \frac{\text{sen}\, u}{\cos^2 u} u' = \text{tg}\, u\ \ \sec u\ \ u' = \text{sen}\, u\ \ \sec^2 u\ \ u'$
40.	$y = \text{cosec}\, x$	$y' = -\frac{\cos x}{\text{sen}^2 x} = -\text{cotg}\, x\ \ \text{cosec}\, x = -\cos x\ \ \text{cosec}^2 x$
41.	$y = \text{cosec}\, u$	$y' = -\frac{\cos u}{\text{sen}^2 u} u' = -\text{cotg}\, u\ \ \text{cosec}\, u u' = -\cos u\ \ \text{cosec}^2 u u'$
42.	$y = \text{arc sen}\, x$	$y' = \frac{1}{\sqrt{1 - x^2}}$
43.	$y = \text{arc sen}\, u$	$y' = \frac{u'}{\sqrt{1 - u^2}}$ (cuando $-\frac{\pi}{2} < \text{arc sen}\, u < \frac{\pi}{2}$)

44.	$y = \text{arc cos } x$	$y' = -\dfrac{1}{\sqrt{1-x^2}}$
45.	$y = \text{arc cos } u$	$y' = -\dfrac{u'}{\sqrt{1-u^2}}$ (cuando $0 < \text{arc cos } u <$
46.	$y = \text{arc tg } x$	$y' = \dfrac{1}{1+x^2}$
47.	$y = \text{arc tg } u$	$y' = \dfrac{u'}{1+u^2}$
48.	$y = \text{arc cotg } x$	$y' = \dfrac{1}{1+x^2} = -\dfrac{1}{1+x^2}$
49.	$y = \text{arc cotg } u$	$y' = \dfrac{u'}{1+u^2} = -\dfrac{1}{1+u^2}$
50.	$y = \text{arc sec } x$	$y' = \dfrac{1}{x\sqrt{x^2-1}}$
51.	$y = \text{arc sec } u$	$y' = \dfrac{u'}{u\sqrt{u^2-1}}$
52.	$y = \text{arc cosec } x$	$y' = -\dfrac{1}{x\sqrt{x^2-1}}$
53.	$y = \text{arc cosec } u$	$y' = -\dfrac{u'}{u\sqrt{u^2-1}}$
54.	$y = \text{sh } x$	$y' = \text{ch } x$
55.	$y = \text{sh } u$	$y' = \text{ch } u\, u'$
56.	$y = \text{ch } x$	$y' = \text{sh } x$
57.	$y = \text{ch } u$	$y' = \text{sh } u\, u'$
58.	$y = \text{t h } x$	$y' = \dfrac{1}{\text{ch}^2 x} = \text{sec h}^2 x$
59.	$y = \text{t h } u$	$y' = \dfrac{u'}{\text{ch}^2 u} = \text{sec h}^2 u \;\; u'$
60.	$y = \text{cot h } x$	$y' = -\dfrac{1}{\text{sh}^2 x} = -\text{cosec h}^2 x$
61.	$y = \text{cot h } u$	$y' = \dfrac{u'}{\text{sh}^2 u}$

62.	$y = \sec h\, x$	$y' = -\dfrac{\text{sh}\, x}{\text{ch}^2 x} = -\sec h\, x \;\; \text{th}\, x$
63.	$y = \sec h\, u$	$y' = -\dfrac{\text{sh}\, u}{\text{ch}^2 u} u' = -\sec h\, u \;\; \text{th}\, u u'$
64.	$y = \text{cosec h}\, x$	$y' = -\dfrac{\text{ch}\, x}{\text{sh}^2 x} = -\text{cosec h}\, x \;\; \cot h\, x$
65.	$y = \text{cosec h}\, u$	$y' = -\dfrac{\text{ch}\, u}{\text{sh}^2 u} u' = -\text{cosec h}\, u \;\; \cot h\, u u'$
66.	$y = \arg \text{sh}\, x$	$y' = \dfrac{1}{\sqrt{x^2 + 1}}$
67.	$y = \arg \text{sh}\, u$	$y' = \dfrac{1}{\sqrt{u^2 + 1}} u'$
68.	$y = \arg \text{ch}\, x$	$y' = \dfrac{1}{\sqrt{x^2 - 1}}$
69.	$y = \arg \text{ch}\, u$	$y' = \dfrac{1}{\sqrt{u^2 - 1}} u'$
70.	$y = \arg \text{th}\, x$	$y' = \dfrac{1}{1 - x^2}$
71.	$y = \arg \text{th}\, u$	$y' = \dfrac{1}{1 - u^2} u'$
72.	$y = \arg \cot h\, x$	$y' = -\dfrac{1}{x^2 - 1}$
73.	$y = \arg \cot h\, u$	$y' = -\dfrac{1}{u^2 - 1} u'$
74.	$y = \arg \sec h\, x$	$y' = -\dfrac{1}{x} \cdot \dfrac{1}{\sqrt{1 - x^2}}$
75.	$y = \arg \sec h\, u$	$y' = -\dfrac{1}{u} \cdot \dfrac{1}{\sqrt{1 - u^2}} u'$
76.	$y = \arg \text{cosec h}\, x$	$y' = -\dfrac{1}{x} \cdot \dfrac{1}{\sqrt{x^2 + 1}}$
77.	$y = \arg \text{cosec h}\, u$	$y' = -\dfrac{1}{u} \cdot \dfrac{1}{\sqrt{u^2 + 1}} u'$

B. TABLA DE INTEGRALES INDEFINIDAS

De [*I*], (página 1) obtenemos la diferencial

$$dy = f'(x)\,dx = F(x)\,dx$$

La integral es $I = \int dy = \int F(x)\,dx$, siendo la función primitiva $f(x) + C$, tal que $f'(x) = F(x)$.

B_1. REGLAS PRINCIPALES PARA LA INTEGRACIÓN

a. Si $f'(x) = F(x)$, entonces

$$\int F(x)\,dx = f(x) + C$$

donde C es una constante arbitraria, llamada constante de integración.

b. Si k es una constante arbitraria, se cumple

$$\int k\,F(x)\,dx = k \int F(x)\,dx$$

c. Se cumple que $\int [F_1(x) \pm F_2(x) \pm F_3(x) \pm \ldots]\,dx = \int F_1(x)\,dx \pm \int F_2(x)\,dx \pm \int F_3(x)\,dx \pm$

d. Si $\int F(x)\,dx = f(x) + C$ y $u = \varphi(x)$, resulta $\int F(u)\,du = f(u) + C$

B_2. TABLA DE INTEGRALES INMEDIATAS

1. $\int dx = x + C$

2. $\int k\,dx = k\,x + C$

3. $\int x^m dx = \dfrac{x^{m+1}}{m+1} + C$ [donde m (real) $\neq -1$], si $m = -1$, resulta:

4. $\int \dfrac{dx}{x} = \ln|x| + C$

5. $\int \dfrac{dF(x)}{F(x)} = \ln|F(x)| + C$

6. $\int \frac{dx}{2\sqrt{x}} = \sqrt{x} + C$

7. $\int \frac{d\ F(x)}{2\sqrt{F(x)}} = \sqrt{F(x)} + C$

8. $\int k^x dx = \frac{k^x}{\ln|k|} + C \quad (\text{con } k > 0 \text{ y } k \neq 1)$

9. $\int k^{ax} dx = \frac{ka^x}{a \ln|k|} + C \quad (\text{con } k > 0 \text{ y } k \neq 1)$

10. $\int k^x \ \ln a \ \ dx = k^x + C$

11. $\int e^x \ dx = e^x + C$

12. $\int e^{ax} \ dx = \frac{e^{ax}}{a} + C$

13. $\int x \ e^{ax} \ dx = \frac{e^{ax}}{a^2}(ax - 1) + C = \frac{e^{ax}}{a}(x - \frac{1}{a}) + C$

14. $\int x \ e^{ax} \ dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1} \ e^{ax} \ dx$

15. $\int \frac{1}{e^x + 1} dx = x - \ln|1 + e^x| + C$

16. $\int \ln x \ dx = x \ \ln|x| - x + C = x(\ln|x| - 1) + C$

17. $\int x \ \ln x \ dx = \frac{x^2}{2}(\ln|x| - \frac{1}{2}) + C$

18. $\int x^m \ \ln x \ dx = \frac{x^{m+1}}{m+1}(\ln|x| - \frac{1}{m+1}) + C,$ si $m = -1$ resulta:

19. $\int \frac{\ln x}{x} dx = \frac{1}{2}\ln^2|x| + C$

20. $\int \operatorname{sen} x \ dx = -\cos x + C$

21. $\int -\operatorname{sen} x \ dx = -\int \operatorname{sen} x \ dx = \cos x + C$

22. $\int \operatorname{sen}^2 x \ dx = \frac{1}{2}(x - \operatorname{sen} x \ \cos x) + C = \frac{x}{2} - \frac{\operatorname{sen} 2x}{4} + C$

23. $\int \operatorname{sen}^n x \ dx = -\frac{\operatorname{sen}^{n-1} x \ \cos x}{n} + \frac{n-1}{n}\int \operatorname{sen}^{n-2} x \ dx$

24. $\int \cos x \; dx = \operatorname{sen} x + C$

25. $\int -\cos x \; dx = -\int \cos x \; dx = -\operatorname{sen} x + C$

26. $\int \cos^2 x \; dx = \frac{1}{2}(x + \operatorname{sen} x \; \cos x) + C = \frac{x}{2} + \frac{\operatorname{sen} 2x}{4} + C$

27. $\int \cos^n x \; dx = \frac{\cos^{n-1} x \; \operatorname{sen} x}{n} + \frac{n-1}{n}\int \cos^{n-2} x \; dx$

28. $\int \frac{1}{\operatorname{sen} x} dx = \ln\left|\operatorname{tg}\frac{x}{2}\right| + C = \ln|\operatorname{cosec} x - \operatorname{cotg} x| + C$

29. $\int \frac{1}{\operatorname{sen}^2 x} dx = -\operatorname{cotg} x + C$

30. $\int -\frac{1}{\operatorname{sen}^2 x} dx = -\int \frac{1}{\operatorname{sen}^2 x} dx = \operatorname{cotg} x + C$

31. $\int \frac{1}{\operatorname{sen}^n x} dx = \frac{-\cos x}{(n-1)\operatorname{sen}^{n-1} x} + \frac{n-2}{n-1}\int \frac{1}{\operatorname{sen}^{n-2} x} dx; \; (n > 1)$

32. $\int \frac{1}{\cos x} dx = \ln\left|\operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C = \ln|\operatorname{tg} x + \sec x| + C$

33. $\int \frac{1}{\cos^2 x} dx = \operatorname{tg} x + C$

34. $\int \frac{1}{\cos^n x} dx = \frac{\operatorname{sen} x}{(n-1)\cos^{n-1} x} + \frac{n-2}{n-1}\int \frac{1}{\cos^{n-2} x} dx; \; (n > 1)$

35. $\int \frac{1}{1 + \cos x} dx = \operatorname{tg}\frac{x}{2} + C$

36. $\int \frac{1}{\operatorname{sen} x \; \cos x} dx = \ln|\operatorname{tg} x| + C$

37. $\int \frac{\operatorname{sen} x}{\cos^2 x} dx = \sec x + C$

38. $\int -\frac{\cos x}{\operatorname{sen}^2 x} dx = -\int \frac{\cos x}{\operatorname{sen}^2 x} dx = \operatorname{cosec} x + C$

39. $$\int \operatorname{sen}^n x \; \cos^m x \; dx = \frac{\operatorname{sen}^{n+1} x \; \cos^{m-1} x}{m+n} + \frac{m-1}{m+n}\int \operatorname{sen}^n x \; \cos^{m-2} x \; dx =$$
$$= -\frac{\operatorname{sen}^{n-1} x \; \cos^{m+1} x}{m+n} + \frac{n-1}{m+n}\int \operatorname{sen}^{n-2} x \; \cos^m x \; dx$$

40. $$\int \frac{\text{sen}^n x}{\cos^m x}dx = -\frac{\text{sen}^{n-1}x}{(n-m)\cos^{m-1}x} + \frac{n-1}{n-m}\int \frac{\text{sen}^{n-2}x}{\cos^m x}dx =$$
$$= \frac{\text{sen}^{n-1}x}{(m-1)\cos^{m-1}x} + \frac{n-m+2}{m-1}\int \frac{\text{sen}^n x}{\cos^{m-2}x}dx =$$
$$= \frac{\text{sen}^{n-1}x}{(m-1)\cos^{m-1}x} - \frac{n-1}{m-1}\int \frac{\text{sen}^{n-2}x}{\cos^{m-2}x}dx$$

41. $$\int \frac{\cos^n x}{\text{sen}^m x}dx = \frac{\cos^{n+1}x}{(m-1)\,\text{sen}^{m-1}x} - \frac{n-m+2}{m-1}\int \frac{\cos^n x}{\text{sen}^{n-2}x}dx =$$
$$= \frac{\cos^{n-1}x}{(n-m)\,\text{sen}^{m-1}x} + \frac{n-1}{n-m}\int \frac{\cos^{n-2}x}{\text{sen}^m x}dx =$$
$$= \frac{\cos^{n-1}x}{(m-1)\,\text{sen}^{m-1}x} - \frac{n-1}{m-1}\int \frac{\cos^{n-2}x}{\text{sen}^{m-2}x}dx$$

42. $$\int x\ \text{sen}\,x\ dx = \text{sen}\,x - x\ \cos x + C$$

43. $$\int x\ \cos x\ dx = \cos x + x\ \text{sen}\,x + C$$

44. $$\int \text{sen}\,nx\ \text{sen}\,mx\ dx = \frac{1}{2}\frac{\text{sen}\,(n-m)x}{n-m} - \frac{1}{2}\frac{\text{sen}\,(n+m)x}{n+m} + C$$

45. $$\int \text{sen}\,nx\ \cos mx\ dx = -\frac{1}{2}\frac{\cos\,(n-m)x}{n-m} - \frac{1}{2}\frac{\cos\,(n+m)x}{n+m} + C$$

46. $$\int \cos nx\ \cos mx\ dx = \frac{1}{2}\frac{\text{sen}\,(n+m)x}{n+m} + \frac{1}{2}\frac{\text{sen}\,(n-m)x}{n-m} + C$$

47. $$\int e^{ax}\ \text{sen}\,nx\ dx = \frac{e^{ax}(a\ \text{sen}\,nx - n\ \cos nx)}{a^2+n^2} + C$$

48. $$\int e^{ax}\ \cos nx\ dx = \frac{e^{ax}(a\ \cos nx + n\ \text{sen}\,nx)}{a^2+n^2} + C$$

49. $$\int \text{tg}\,x\ dx = -\ln|\cos x| + C = \ln|\sec x| + C$$

50. $$\int \text{tg}^2 x\ dx = \text{tg}\,x - x + C$$

51. $$\int \text{tg}^n x\ dx = \frac{\text{tg}^{n-1}x}{n-1} - \int \text{tg}^{n-2}x\ dx$$

52. $$\int \text{cotg}\,x\ dx = \ln|\text{sen}\,x| + C$$

53. $\int \operatorname{cotg}^2 x \; dx = -\operatorname{cotg} x - x$

54. $\int \operatorname{cotg}^n x \; dx = -\dfrac{\operatorname{cotg}^{n-1} x}{n-1} - \int \operatorname{cotg}^{n-2} x \; dx$

55. $\int \dfrac{1}{\operatorname{tg} x} \; dx = \ln|\operatorname{sen} x| + C$

56. $\int \dfrac{1}{\operatorname{cotg} x} dx = -\ln|\cos x| + C$

57. $\int x \; \operatorname{tg}^2 x \; dx = x \; \operatorname{tg} x + \ln|\cos x| - \dfrac{x^2}{2} + C$

58. $\int x \; \operatorname{cotg}^2 x \; dx = x \; \operatorname{cotg} x + \ln|\operatorname{sen} x| - \dfrac{x}{2} + C$

59. $\int \sec x \; dx = \int \dfrac{1}{\cos x} dx = \ln|\sec x + \operatorname{tg} x| + C = \ln \; \operatorname{tg} \left(\dfrac{x}{2} + \dfrac{\pi}{4}\right) + C$

60. $\int \sec^2 x \; dx = \operatorname{tg} x$

61. $\int \sec^n x \; dx = \int \dfrac{1}{\cos x} dx = \dfrac{\operatorname{sen} x}{(n-1)\cos^{n-1} x} + \dfrac{n-2}{n-1} \int \sec^{n-2} x \; dx =$

$$= \frac{\sec^{n-2} x \; \operatorname{tg} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \; dx$$

62. $\int x \; \sec^2 x \; dx = x \; \operatorname{tg} x + \ln|\cos x|$

63. $\int \dfrac{dx}{\sec x} = \operatorname{sen} x + C$

64. $\int \operatorname{cosec} x \; dx = \int \dfrac{1}{\operatorname{sen} x} dx = \ln|\operatorname{cosec} x - \operatorname{cotg} x| + C = \ln \left|\operatorname{tg} \dfrac{x}{2}\right| + C$

65. $\int \operatorname{cosec}^2 x \; dx = -\operatorname{cotg} x + C$

66. $\int \dfrac{dx}{\operatorname{cosec} x} = -\cos x + C$

67. $\int \operatorname{cosec}^n x \; dx = \int \dfrac{1}{\operatorname{sen}^n x} dx = \dfrac{-\cos x}{(n-1)\operatorname{sen}^{n-1} x} +$

$$+ \frac{n-2}{n-1} \int \frac{1}{\operatorname{sen}^{n-2} x} dx = -\frac{\operatorname{cosec}^{n-2} x}{n-1} +$$

$$+ \frac{n-2}{n-1} \int \operatorname{cosec}^{n-2} x \; dx$$

68. $\int x \ \operatorname{cosec}^2 x \ dx = -x \ \operatorname{cotg} x + \ln|\operatorname{sen} x| + C$

69. $\int \operatorname{arc\,sen} x \ dx = x \ \operatorname{arc\,sen} x + \sqrt{1-x^2} + C$

70. $\int x \ \operatorname{arc\,sen} x \ dx = (\frac{x^2}{2} - \frac{1}{4}) \operatorname{arc\,sen} x + \frac{x\sqrt{1-x^2}}{4} + C$

71. $\int x^2 \ \operatorname{arc\,sen} x \ dx = \frac{x^3}{3} \operatorname{arc\,sen} x + \frac{(x^2+2)\sqrt{1-x^2}}{9} + C$

72. $\int x^n \ \operatorname{arc\,sen} x \ dx = \frac{x^{n+1}}{n+1} \operatorname{arc\,sen} x - \frac{1}{n+1} \int \frac{x^{n+1}}{\sqrt{1-x^2}} dx$

73. $\int \operatorname{arc\,cos} x \ dx = x \ \operatorname{arc\,cos} x - \sqrt{1-x^2} + C$

74. $\int x \ \operatorname{arc\,cos} x \ dx = (\frac{x^2}{2} - \frac{1}{4}) \operatorname{arc\,cos} x - \frac{x\sqrt{1-x^2}}{4} + C$

75. $\int x^2 \ \operatorname{arc\,cos} x \ dx = \frac{x^3}{3} \operatorname{arc\,cos} x - \frac{(x^2+2)\sqrt{1-x^2}}{9} + C$

76. $\int x^n \ \operatorname{arc\,cos} x \ dx = \frac{x^{n+1}}{n+1} \operatorname{arc\,cos} x + \frac{1}{n+1} \int \frac{x^{n+1}}{\sqrt{1-x^2}} dx$

77. $\int \operatorname{arc\,tg} x \ dx = x \ \operatorname{arc\,tg} x - \frac{1}{2} \ln(1+x^2) + C$

78. $\int x \ \operatorname{arc\,tg} x \ dx = \frac{1}{2}(x^2+1) \operatorname{arc\,tg} x - \frac{x}{2} + C$

79. $\int x^2 \ \operatorname{arc\,tg} x \ dx = \frac{x^3}{3} \operatorname{arc\,tg} x - \frac{x^2}{6} + \frac{1}{6} \ln|1+x^2| + C$

80. $\int x^n \ \operatorname{arc\,tg} x \ dx = \frac{x^{n+1}}{n+1} \operatorname{arc\,tg} x - \frac{1}{n+1} \int \frac{x^{n+1}}{1+x^2} dx$

81. $\int \operatorname{arc\,cotg} x \ dx = x \ \operatorname{arc\,cotg} x + \frac{1}{2} \ln|x^2+1| + C$

82. $\int x \ \operatorname{arc\,cotg} x \ dx = \frac{1}{2}(x^2+1) \operatorname{arc\,cotg} x + \frac{x}{2} + C$

83. $\int x^2 \ \operatorname{arc\,cotg} x \ dx = \frac{x^3}{3} \operatorname{arc\,cotg} x + \frac{x^2}{6} - \frac{1}{6} \ln|x^2+1| + C$

84. $\int x^n \operatorname{arc\,cotg} x \; dx = \frac{x^{n+1}}{n+1} \operatorname{arc\,cotg} x + \frac{1}{n+1} \int \frac{x^{n+1}}{1+x^2} dx$

85. $\int \operatorname{arc\,sec} x \; dx = x \operatorname{arc\,sec} x - \ln\left|x + \sqrt{x^2 - 1}\right| + C; \; (0 < \operatorname{arc\,sec} x < \frac{\pi}{2}) =$

$= x \operatorname{arc\,sec} x + \ln\left|x + \sqrt{x^2 - 1}\right| + C; \; (\frac{\pi}{2} < \operatorname{arc\,sec} x < \pi)$

86. $\int x \operatorname{arc\,sec} x \; dx = \frac{x^2}{2} \operatorname{arc\,sec} x - \frac{\sqrt{x^2 - 1}}{2} + C; \; (0 < \operatorname{arc\,sec} x < \frac{\pi}{2}) =$

$= \frac{x^2}{2} \operatorname{arc\,sec} x + \frac{\sqrt{x^2 - 1}}{2} + C; \; (\frac{\pi}{2} < \operatorname{arc\,sec} x < \pi)$

87. $\int x^2 \operatorname{arc\,sec} x \; dx = \frac{x^3}{3} \operatorname{arc\,sec} x - \frac{x\sqrt{x^2 - 1}}{2} - \frac{1}{6} \ln\left|\sqrt{x^2 - 1}\right| + C; \; (0 < \operatorname{arc\,sec} x < \frac{\pi}{2})$

$= \frac{x^3}{3} \operatorname{arc\,sec} x + \frac{x\sqrt{x^2 - 1}}{2} + \frac{1}{6} \ln\left|\sqrt{x^2 - 1}\right| + C; \; (\frac{\pi}{2} < \operatorname{arc\,sec} x < \pi)$

88. $\int x^n \operatorname{arc\,sec} x \; dx = \frac{x^{n+1}}{n+1} \operatorname{arc\,sec} x - \frac{1}{n+1} \int \frac{x^n \; dx}{\sqrt{x^2 - 1}}; \; (0 < \operatorname{arc\,sec} x < \frac{\pi}{2}) =$

$= \frac{x^{n+1}}{n+1} \operatorname{arc\,sec} x + \frac{1}{n+1} \int \frac{x^n \; dx}{\sqrt{x^2 - 1}}; \; (\frac{\pi}{2} < \operatorname{arc\,sec} x < \pi)$

89. $\int \operatorname{arc\,cosec} x \; dx = x \operatorname{arc\,cosec} x + \ln\left|(x + \sqrt{x^2 - 1})\right| + C; \; (0 < \operatorname{arc\,cosec} x < \frac{\pi}{2}) =$

$= x \operatorname{arc\,cosec} x - \ln\left|(x + \sqrt{x^2 - 1})\right| + C; \; (-\frac{\pi}{2} < \operatorname{arc\,cosec} x < 0)$

90. $\int x \operatorname{arc\,cosec} x \; dx = \frac{x^2}{2} \operatorname{arc\,cosec} x + \frac{\sqrt{x^2 - 1}}{2} + C; \; (0 < \operatorname{arc\,cosec} x < \frac{\pi}{2}) =$

$= \frac{x^2}{2} \operatorname{arc\,cosec} x - \frac{\sqrt{x^2 - 1}}{2} + C; \; (-\frac{\pi}{2} < \operatorname{arc\,cosec} x < 0)$

91. $\int x^2 \operatorname{arc\,cosec} x \; dx = x + \frac{x\sqrt{x^2 - 1}}{2} + \frac{1}{6} \ln\left|x + \sqrt{x^2 - 1}\right| + C; \; (0 < \operatorname{arc\,cosec} x < \frac{\pi}{2}) =$

$= x - \frac{x\sqrt{x^2 - 1}}{2} - \frac{1}{6} \ln\left|x + \sqrt{x^2 - 1}\right| + C; \; (-\frac{\pi}{2} < \operatorname{arc\,cosec} x < 0)$

92. $$\int x^n \operatorname{arc\,cosec} x \; dx = \frac{x^{n+1}}{n+1} \operatorname{arc\,cosec} x + \frac{1}{n+1} \int \frac{x^n \; dx}{\sqrt{x^2-1}} + C; \; (0 < \operatorname{cosec} x < \frac{\pi}{2}) =$$
$$= \frac{x^{n+1}}{n+1} \operatorname{arc\,cosec} x - \frac{1}{n+1} \int \frac{x^n \; dx}{\sqrt{x^2-1}} + C; \; (\frac{\pi}{2} - \operatorname{arc\,cosec} x < \pi)$$

93. $$\int \operatorname{sh} x \; dx = \operatorname{ch} x + C$$

94. $$\int x \; \operatorname{sh} x \; dx = -\operatorname{sh} x + x \; \operatorname{ch} x + C$$

95. $$\int \operatorname{sh}^2 x \; dx = -\frac{x}{2} + \frac{\operatorname{sh} x \; \operatorname{ch} x}{2} + C$$

96. $$\int \operatorname{ch} x \; dx = \operatorname{sh} x + C$$

97. $$\int x \; \operatorname{ch} x \; dx = -\operatorname{ch} x + x \; \operatorname{sh} x + C$$

98. $$\int \operatorname{ch}^2 x \; dx = \frac{x}{2} + \frac{\operatorname{sh} x \; \operatorname{ch} x}{2} + C$$

99. $$\int \operatorname{sh} x \; \operatorname{ch} x \; dx = \frac{\operatorname{sh}^2 x}{2}$$

100. $$\int \frac{1}{\operatorname{sh} x \; \operatorname{ch} x} dx = \ln |\operatorname{th} x| + C$$

101. $$\int \operatorname{th} x \; dx = \ln |\operatorname{ch} x| + C$$

102. $$\int \operatorname{th}^2 x \; dx = -\operatorname{th} x + x + C$$

103. $$\int \operatorname{coth} x \; dx = \ln |\operatorname{sh} x| + C$$

104. $$\int \operatorname{coth}^2 x \; dx = -\operatorname{coth} x + x + C$$

105. $$\int \operatorname{sech} x \; dx = 2 \operatorname{arc\,tg} e^x + C$$

106. $$\int \operatorname{sech}^2 x \; dx = \operatorname{tgh} x + C$$

107. $$\int \operatorname{cosech} x \; dx = \ln \left| \operatorname{tgh} \frac{x}{2} \right| + C$$

108. $$\int \operatorname{arg\,sh} x \; dx = x \; \operatorname{arg\,sh} - \sqrt{1+x^2} + C$$

109. $$\int x \; \operatorname{arg\,sh} x \; dx = (\frac{x^2}{2} + \frac{1}{4}) \operatorname{arg\,sh} x - \frac{x\sqrt{1+x^2}}{4} + C$$

110. $\int \arg\text{ch}\,x \ dx = x \ \arg\text{ch}\,x - \sqrt{x^2-1} + C; \ (\arg\text{ch}\,x > 0) =$

$= x \ \arg\text{ch}\,x + \sqrt{x^2-1} + C; \ (\arg\text{ch}\,x < 0)$

111. $\int x \ \arg\text{ch}\,x \ dx = (\frac{x^2}{2} - \frac{1}{4}) \arg\text{ch}\,x - \frac{x\sqrt{x^2-1}}{4} + C; \ (\arg\text{ch}\,x > 0) =$

$= (\frac{x^2}{2} - \frac{1}{4}) \arg\text{ch}\,x + \frac{x\sqrt{x^2-1}}{4} + C; \ (\arg\text{ch}\,x < 0)$

112. $\int \arg\text{th}\,x \ dx = x \ \arg\text{th}\,x + \frac{1}{2}\ln|(1-x^2)| + C$

113. $\int x \ \arg\text{th}\,x \ dx = \frac{1}{2}(x^2-1)\arg\text{th}\,x + \frac{x}{2} + C$

114. $\int \arg\text{coth}\,x \ dx = x \ \arg\text{ch}\,x + \frac{1}{2}\ln|(x^2-1)| + C$

115. $\int x \ \arg\text{coth}\,x \ dx = \frac{1}{2}(x^2-1)\arg\text{coth}\,x + \frac{x}{2} + C$

116. $\int \arg\text{sech}\,x \ dx = x \ \arg\text{sech}\,x + \arg\text{sh}\,x + C; \ (\arg\text{sech}\,x > 0) =$

$= x \ \arg\text{sech}\,x - \arg\text{sh}\,x + C; \ (\arg\text{sech}\,x < 0)$

117. $\int x \ \arg\text{sech}\,x \ dx = \frac{x^2}{2}\arg\text{sech}\,x - \frac{\sqrt{1-x^2}}{2} + C; \ (\arg\text{sech}\,x > 0) =$

$= \frac{x^2}{2}\arg\text{sech}\,x + \frac{\sqrt{1-x^2}}{2} + C; \ (\arg\text{sech}\,x < 0)$

118. $\int \arg\text{cosech}\,x \ dx = x \ \arg\text{cosech}\,x \pm \arg\text{sh}\,x + C; \ (+\text{si}\,x > 0, -\text{si}\,x < 0)$

119. $\int x \ \arg\text{cosech}\,x \ dx = \frac{x^2}{2}\arg\text{cosech}\,x \pm \frac{\sqrt{x^2+1}}{2} + C; \ (+\text{si}\,x > 0, -\text{si}\,x < 0)$

120. $\int \frac{dx}{\cos\text{h}^2 x} = \text{tgh}\,x + C$

121. $\int -\frac{dx}{\text{senh}^2 x} = \text{cotgh}\,x + C$

122. $\int -\frac{\text{sh}\,x}{\text{ch}^2 x} = \text{sech}\,x + C$

123. $\int -\frac{\text{ch}\,x}{\text{sh}^2 x} = \text{cosech}\,x + C$

124. $\int (x+b)^n dx = \dfrac{(x+b)^{n+1}}{n+1} + C$; (para $n \neq -1$; para $n = -1$ resulta) :

125. $\int \dfrac{dx}{x+b} = \ln|(x+b)| + C$

126. $\int \dfrac{x\ \ dx}{x+b} = x - b\ \ \ln|(ax+b)| + C$

127. $\int \dfrac{dx}{\sqrt{x+b}} = 2\sqrt{x+b} + C$

128. $\int \dfrac{x\ \ dx}{\sqrt{x+b}} = \dfrac{2}{3}(x-2b)\sqrt{x+b} + C$

129. $\int \dfrac{1}{(x+b)\ (x+d)} dx = \dfrac{1}{b-d} \ln\left|\dfrac{x+d}{x+b}\right| + C.\ (b-d \neq 0)$

130. $\int \dfrac{x\ \ dx}{(x+b)\ (x+d)} = \dfrac{1}{b-d}[b\ \ \ln|(x+b)| - d\ \ \ln|(x+d)| + C]$; (si $b - d \neq 0$)

131. $\int \dfrac{x+d}{\sqrt{x+b}} dx = \dfrac{2}{3}(x+3d-2b) + C$

132. $\int \dfrac{dx}{\sqrt{(x+b)\ (x+d)}} = \dfrac{2}{\sqrt{-1}} \operatorname{arc\,tg} \sqrt{\dfrac{-(x+b)}{(x+d)}} + C$

133. $\int \dfrac{dx}{x^2+x+c} = \dfrac{1}{\sqrt{1-4c}} \ln\left|\dfrac{2x+1-\sqrt{1-4c}}{2x+1+\sqrt{1-4c}}\right| + C =$

$$= \frac{1}{\sqrt{4c-1}} \ln\left|\frac{2x+1-\sqrt{4c-1}}{2x+1+\sqrt{4c-1}}\right| + C;\ (4c-1>0) =$$

$$= \frac{2}{\sqrt{1-4c}} \operatorname{arc\,tg} \frac{2x+b}{\sqrt{1-4c}} + C;\ (4c-1<0)$$

134. $\int \dfrac{dx}{\sqrt{x^2+x+c}} = \text{n}\left|\sqrt{x^2+x+c} + 2x + 1\right| + C$

135. $\int \dfrac{dx}{x^2+1} = \operatorname{arc\,tg} x + C$

136. $\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a} \operatorname{arc\,tg} \dfrac{x}{a} + C$

137. $\int \dfrac{x\ \ dx}{x^2+1} = \dfrac{1}{2} \ln|x^2+1| + C$

138. $\int \frac{x^2 dx}{x^2+1} = \operatorname{arc\,tg} x + C$

139. $\int \frac{x^2 dx}{x^2+a^2} = x - \operatorname{arc\,tg} \frac{x}{a} + C$

140. $\int \frac{-1}{x^2+1} dx = \operatorname{arc\,cotg} x + C$

141. $\int \sqrt{x^2+1}\, dx = \frac{x\sqrt{x^2+1}}{2} + \frac{1}{2}\ln\left|(x+\sqrt{x^2+1})\right| + C$

142. $\int \frac{dx}{\sqrt{x^2+1}} = \ln\left|(x+\sqrt{x^2+1})\right| + C = \operatorname{arg\,sh} x + C$

143. $\int \frac{x\ dx}{\sqrt{x^2+1}} = \sqrt{x^2+1} + C$

144. $\int \sqrt{x^2+a^2}\, dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\ln\left|(x+\sqrt{x^2+a^2})\right| + C$

145. $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|(x+\sqrt{x^2+a^2})\right| = \operatorname{arg\,sh} \frac{x}{a} + C$

146. $\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right| + C$

147. $\int \frac{x\ dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2} + C$

148. $\int \frac{dx}{x^2-1} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C;\ (x^2 > 1)$

149. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C;\ (x^2 > a^2)$

150. $\int \frac{x\ dx}{x^2-1} = \frac{1}{2}\ln\left|(x^2-1)\right| + C;\ (x^2 > a^2)$

151. $\int \frac{x\ dx}{x^2-a^2} = \frac{1}{2}\ln\left|(x^2-a^2)\right| + C;\ (x^2 > a^2)$

152. $\int \frac{x^2 dx}{x^2-a^2} = x + \frac{a}{2}\ln\left|\frac{x-a}{x+a}\right| + C;\ (x^2 > a^2)$

153. $\int \frac{dx}{\sqrt{x^2-1}} = \ln\left|(x+\sqrt{x^2-1})\right| + C$

154. $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|(x+\sqrt{x^2-a^2})\right| + C$

155. $\int \frac{x\ dx}{\sqrt{x^2-1}} = \sqrt{x^2-1} + C$

156. $\int \frac{x\ dx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2} + C$

157. $\int \frac{dx}{x\sqrt{x^2-1}} = \text{arc sec}\,|x| + C$

158. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\text{arc sec}\left|\frac{x}{a}\right|$

159. $\int \frac{dx}{1-x^2} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C = \text{arg th}\,x + C;\ (x^2<1)$

160. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C = \frac{1}{a}\text{arg th}\,\frac{x}{a} + C;\ (x^2<a^2)$

161. $\int \frac{x\ dx}{1-x^2} = -\frac{1}{2}\ln\left|(1-x^2)\right| + C;\ (x^2<1)$

162. $\int \frac{x\ dx}{a^2-x^2} = -\frac{1}{2}\ln\left|(a^2-x^2)\right| + C;\ (x^2<a^2)$

163. $\int \frac{dx}{\sqrt{1-x^2}} = \text{arc sen}\,x + C$

164. $\int \frac{dx}{\sqrt{a^2-x^2}} = \text{arc sen}\,\frac{x}{a} + C$

165. $\int \frac{x\ dx}{\sqrt{1-x^2}} = -\sqrt{1-x^2} + C$

166. $\int \frac{x\ dx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2} + C$

167. $\int \frac{dx}{x\sqrt{1-x^2}} = -\ln\left|\frac{\sqrt{1-x^2}}{x}\right| + C$

168. $\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$

Veremos las integrales que se presentan con más frecuencia, según la siguiente clasificación:

B$_3$. INTEGRALES DE FUNCIONES RACIONALES E IRRACIONALES

a. Cuando en las integrales figura la expresión $ax + b$ ó la expresión $\sqrt{ax+b}$

169. $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$; (para $n \neq -1$, para n 1 resulta):

170. $\int \frac{dx}{ax+b} = \frac{1}{a}\ln|x| + C$

171. $\int \frac{dx}{x(ax+b)} = \frac{1}{b}\ln\left|\frac{x}{ax+b}\right| + C$

172. $\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right| + C$

173. $\int \frac{dx}{x^3(ax+b)} = \frac{2ax-b}{2b^2x^2} + \frac{a^2}{b^3}\ln\left|\frac{x}{ax+b}\right| + C$

174. $\int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a} + C$

175. $\int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{b}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + C$; (para $b \neq 0$)

176. $\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}} + C$; (para $b \neq 0$)

177. $\int \frac{dx}{x^n\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(n-1)bx^{n-1}} - \frac{(2n-3)a}{(2n-2)b}\int \frac{dx}{x^{n-1}\sqrt{ax+b}}$; $(n \neq 1)$

178. $\int \frac{dx}{(ax+b)^2} = -\frac{1}{a(ax+b)} + C$

179. $\displaystyle\int\frac{dx}{x\,(ax+b)^2} = \frac{1}{b\,(ax+b)} + \frac{1}{b^2}\ln\left|\frac{x}{ax+b}\right| + C$

180. $\displaystyle\int\frac{dx}{x^2\,(ax+b)^2} = -a\,(\frac{1}{b^2\,(ax+b)} + \frac{1}{ab^2x} - \frac{2}{b^3}\ln\left|\frac{ax+b}{x}\right|) + C$

181. $\displaystyle\int\frac{dx}{x^3\,(ax+b)^2} = -\frac{1}{b^4}\left[3a^2\ln\left|\frac{ax+b}{x}\right| + \frac{a^3x}{ax+b} + \frac{(ax+b)^2}{2x^2} - \frac{3a\,(ax+b)}{x}\right] + C$

182. $\displaystyle\int\frac{dx}{(ax+b)^3} = -\frac{1}{2\,(ax+b)^2} + C$

183. $\displaystyle\int\frac{dx}{x\,(ax+b)^3} = -\frac{1}{b^3}\,(\ln\left|\frac{ax+b}{x}\right| + \frac{2ax}{ax+b} - \frac{a^2x^2}{ax+b}) + C$

184. $\displaystyle\int\frac{dx}{x^2\,(ax+b)^3} = -a\left[\frac{1}{2b^2\,(ax+b)^2} + \frac{2}{b^3\,(ax+b)} + \frac{1}{b^3x} - \frac{3}{b^4}\ln\left|\frac{ax+b}{x}\right|\right] + C$

185. $\displaystyle\int\frac{dx}{x^3\,(ax+b)^3} = -\frac{1}{b^5}\left[6a^2\ln\left|\frac{ax+b}{x} + \frac{4a^3x}{ax+b} - \frac{a4x^2}{2\,(ax+b)^2} + \frac{(ax+b)^2}{2x^2} - \frac{4a\,(ax+b)}{x}\right|\right] + C$

186. $\displaystyle\int\frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int\frac{dx}{x\sqrt{ax+b}}$ (resuelta en 175)

187. $\displaystyle\int\frac{\sqrt{ax+b}}{x^2}\,dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int\frac{dx}{x\sqrt{ax+b}}$

188. $\displaystyle\int\frac{\sqrt{ax+b}}{x^n}\,dx = -\frac{\sqrt{ax+b}}{(n-1)\,x^{n-1}} + \frac{a}{2\,(n-1)}\int\frac{dx}{x^{n-1}\sqrt{ax+b}} \quad (n \neq 1)$

189. $\displaystyle\int\frac{x\;dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln|\,(ax+b)| + C$

190. $\displaystyle\int\frac{x^2dx}{ax+b} = \frac{(ax+b)^2}{2a^3} - \frac{2b\,(ax+b)}{a^3} + \frac{b^2}{a^3}\ln|\,(ax+b)| + C$

191. $\displaystyle\int\frac{x^3dx}{ax+b} = \frac{1}{a^4}\left[\frac{(ax+b)^3}{3} - \frac{3b\,(ax+b)^2}{2} + 3b^2\,(ax+b) - b^3\ln|\,(ax+b)|\right] + C$

192. $\displaystyle\int\frac{x\;dx}{(ax+b)^2} = \frac{b}{a^2\,(ax+b)} + \frac{1}{a^2}\ln|\,(ax+b)| + C$

193. $\displaystyle\int\frac{x^2dx}{(ax+b)^2} = \frac{1}{a^3}\left(ax+b - \frac{b^2}{2\,(ax+b)^2} - 2b\ln|\,(ax+b)|\right) + C$

194. $\int \frac{x^3dx}{(ax+b)^2} = \frac{1}{a^4}\left[\frac{(ax+b)^2}{2} - 3b(ax+b) + 3b^2\ln|(ax+b)| + \frac{b^3}{ax+b}\right] + C$

195. $\int \frac{x\ dx}{(ax+b)^3} = \frac{1}{a^2}\left(-\frac{1}{ax+b} + \frac{b}{2(ax+b)^2}\right) + C$

196. $\int \frac{x^2dx}{(ax+b)^3} = \frac{1}{a^3}\left(\ln|(ax+b)| + \frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2}\right) + C$

197. $\int \frac{x^3dx}{(ax+b)^3} = \frac{1}{a^4}\left(ax+b-3b\ \ln|(ax+b)| - \frac{3b^2}{ax+b} + \frac{b^3}{ax+b}\right) + C$

198. $\int \sqrt{ax+b}\,dx = \frac{2\sqrt{(ax+b)^3}}{3a} + C$

199. $\int x\sqrt{(ax+b)}\,dx = \frac{2(3ax-2b)}{15a^2}\sqrt{(ax+b)^3} + C$

200. $\int x^2\sqrt{ax+b}\,dx = \frac{2(15a^2x^2-2abx+8b^2)}{15a^2}\sqrt{(ax+b)^3} + C$

201. $\int x^n\sqrt{ax+b}\,dx = \frac{2x^n}{(2n+3)a}\sqrt{(ax+b)^3} + C$

202. $\int \frac{x\ dx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b} + C$

203. $\int \frac{x^2dx}{\sqrt{ax+b}} = \frac{2(3a^2x^2-4abx+8b^2)\sqrt{ax+b}}{15a^3} + C$

204. $\int \frac{x^ndx}{\sqrt{ax+b}} = \frac{2x^n\sqrt{ax+b}}{(2n+1)a} - \frac{2nb}{(2n+1)a}\int \frac{x^{n-1}dx}{\sqrt{ax+b}}$

205. $\int \frac{\sqrt{ax+b}}{x}dx = 2\sqrt{ax+b}\int \frac{ax+b}{-b}$

206. $\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}}$

207. $\int \sqrt{(ax+b)^3}\,dx = \frac{2\sqrt{(ax+b)^5}}{5a} + C$

208. $\int \sqrt{(ax+b)^n}\,dx = \frac{2\sqrt{(ax+b)^{n+4}}}{a(n+2)} + C$

426. $\displaystyle\int \frac{dx}{x^3(x^4+a^4)} = -\frac{1}{2a^4x^2} - \frac{1}{2a^6}\operatorname{arc\,tg}\frac{x^2}{a^2} + C$

427. $\displaystyle\int \frac{dx}{x(x^4-a^4)} = \frac{1}{4a^4}\ln\left|\frac{x^4-a^4}{x^4}\right| + C$

428. $\displaystyle\int \frac{x\ dx}{x^3+a^3} = \frac{1}{6a}\ln\left|\frac{x^2-ax+a^2}{(x+a)^2}\right| + \frac{1}{a\sqrt{3}}\operatorname{arc\,tg}\frac{2x-a}{a\sqrt{3}} + C$

429. $\displaystyle\int \frac{x\ dx}{x^4+a^4} = \frac{1}{2a^2}\operatorname{arc\,tg}\frac{x^2}{a^2} + C$

430. $\displaystyle\int \frac{x\ dx}{x^4-a^4} = \frac{1}{4a^2}\ln\left|\frac{x^2-a^2}{x^2+a^2}\right| + C$

431. $\displaystyle\int \frac{x^2dx}{x^3+a^3} = \frac{1}{3}\ln|x^3+a^3| + C$

432. $\displaystyle\int \frac{x^2dx}{x^3-a^3} = \frac{1}{3}\ln|x^3-a^3| + C$

433. $\displaystyle\int \frac{x^2dx}{x^4+a^4} = \frac{1}{2a^2}\operatorname{arc\,tg}\frac{x^2}{a^2} + C$

434. $\displaystyle\int \frac{x^2dx}{x^4-a^4} = \frac{1}{4a}\ln\left|\frac{x-a}{x+a}\right| + \frac{1}{2a}\operatorname{arc\,tg}\frac{x}{a} + C$

435. $\displaystyle\int \frac{x^3dx}{x^4+a^4} = \frac{1}{4}\ln|x^4+a^4| + C$

436. $\displaystyle\int \frac{x^3dx}{x^4-a^4} = \frac{1}{4}\ln|x^4-a^4| + C$

437. $\displaystyle\int \frac{x\ dx}{(x^3+a^3)^2} = \frac{x^2}{3a^3(x^3+a^3)} + \frac{1}{18a^4}\ln\left|\frac{x^2-ax+a^2}{(x+a)^2}\right| + \frac{1}{3a^4\sqrt{3}}\operatorname{arc\,tg}\frac{2x-a}{a\sqrt{3}} + C$

438. $\displaystyle\int \frac{x^2dx}{(x^3+a^3)^2} = -\frac{1}{3(x^3+a^3)} + C$

439. $\displaystyle\int \frac{dx}{x(x^3+a^3)^2} = \frac{1}{3a^3(x^3+a^3)} + \frac{1}{3a^6}\ln\left|\frac{x^3}{x^3+a^3}\right| + C$

440. $\displaystyle\int \frac{dx}{x^2(x^3+a^3)^2} = -\frac{1}{a^6x} - \frac{x^2}{3a^6(x^3+a^3)} - \frac{4}{3a^6}\int\frac{x\ dx}{x^3+a^3} + C$; (ver ejercicio 428)

441. $\int \frac{dx}{x^2(x^4-a^4)} = \frac{1}{a^4 x} + \frac{1}{4a^6}\ln\left|\frac{x-a}{x+a}\right| + \frac{1}{2a^6}\operatorname{arc\,tg}\frac{x}{a} + C$

442. $\int \frac{dx}{x^3(x^4-a^4)} = \frac{1}{2a^4x^2} + \frac{1}{4a^6}\ln\left|\frac{x^2-a^2}{x^2+a^2}\right| + C$

443. $\int \frac{dx}{x\sqrt{x^n-a^n}} = \frac{2}{n\sqrt{a^n}}\arccos\sqrt{\frac{a^n}{x^n}} + C$

444. $\int \frac{dx}{x^n(x^3+a^3)} = -\frac{1}{a^3(n-1)x^{n-1}} - \frac{1}{a^3}\int\frac{dx}{x^{n-3}(x^3+a^3)}$

445. $\int \frac{x^n dx}{x^3+a^3} = \frac{x^{n-2}}{(n-2)} - a^3\int\frac{x^{n-3}dx}{x^3+a^3}$

446. $\int \frac{x^{n-1}dx}{x^n+a^n} = \frac{1}{n}\ln|x^n+a^n| + C$

447. $\int \frac{x^{n-1}dx}{x^n-a^n} = \frac{1}{n}\ln|x^n-a^n| + C$

h. Cuando en las integrales figura $\sqrt{x}$,ó las expresiones $\sqrt{x}$ y $a^2 \pm b^2x$

448. $\int \frac{dx}{(a^4+x^2)\sqrt{x}} = \frac{1}{2a^3\sqrt{2}}\ln\left|\frac{x+a\sqrt{2x}+a^2}{x-a\sqrt{2x}+a^2}\right| + \frac{1}{a^3\sqrt{2}}\operatorname{arc\,tg}\frac{a\sqrt{2x}}{a^2-x} + C$

449. $\int \frac{dx}{(a^4-x^2)\sqrt{x}} = \frac{1}{2a^3}\ln\left|\frac{a+\sqrt{x}}{a-\sqrt{x}}\right| + \frac{1}{a^2}\operatorname{arc\,tg}\frac{\sqrt{x}}{a} + C$

450. $\int \frac{\sqrt{x}dx}{a^4+x^2} = -\frac{1}{2a\sqrt{2}}\ln\frac{x+a\sqrt{2x}+a^2}{x-a\sqrt{2x}+a^2} + \frac{1}{a\sqrt{2}}\operatorname{arc\,tg}\frac{a\sqrt{2x}}{a^2-x} + C$

451. $\int \frac{\sqrt{x}dx}{a^4-x^2} = \frac{1}{2a}\ln\frac{a+\sqrt{x}}{a-\sqrt{x}} - \frac{1}{a}\operatorname{arc\,tg}\frac{\sqrt{x}}{a}$

452. $\int \frac{dx}{(a^2+b^2x)\sqrt{x}} = \frac{2}{ab} + \operatorname{arc\,tg}\frac{b\sqrt{x}}{a} + C$

453. $\int \frac{dx}{(a^2-b^2x)\sqrt{x}} = \frac{2}{ab}\left[\frac{1}{2}\ln\left|\frac{a+b\sqrt{x}}{a-b\sqrt{x}}\right|\right] + C$

$$+ a(m+n-2)\int \frac{dx}{(ax+b)^m (cx+d)^n}\Big]$$

232. $$\int \frac{dx}{(cx+d)^n\sqrt{ax+b}} = \frac{\sqrt{ax+b}}{(n-1)\,(ad-bc)\,(cx+d)^{n-1}} + \frac{(2n-3)\,a}{2\,(n-1)\,(ad-bc)}\int \frac{dx}{(cx+d)^{n-1}\sqrt{ax+b}}$$

233. $$\int \frac{(cx+d)^n}{\sqrt{ax+b}}dx = \frac{2\,(cx+d)^n\sqrt{ax+b}}{(2n+1)\,a} + \frac{2n\,(ad-bc)}{(2n+1)\,a}\int \frac{cx+d^{n-1}}{\sqrt{ax+b}}dx$$

234. $$\int \sqrt{ax+b}\,(cx+d)^n dx = 2\sqrt{ax+b}\,(cx+d)^{n+1} + (bc-ad)\int \frac{(cx+d)^n dx}{\sqrt{ax+b}}$$ (ver ejercicio 233)

235. $$\int \frac{dx}{\sqrt{(ax+b)\,(cx+d)^n}} = -\frac{1}{(n-1)\,(bc-ad)}\left[\frac{\sqrt{ax+b}}{(cx+d)^{n-1}} + (n-\frac{3}{2})\,a\,\right]\int \frac{dx}{\sqrt{ax+b}\,(cx+d)^{n-1}}$$

236. $$\int \frac{\sqrt{ax+b}}{(cx+d)^n}dx = \frac{-\sqrt{ax+b}}{(n-1)\,c\,(cx+d)^{n-1}} + \frac{a}{2\,(n-1)\,c}\int \frac{dx}{(cx+d)^{n-1}\sqrt{ax+b}}$$

237. $$\int \frac{\sqrt{ax+b}dx}{cx+d} = \frac{2\sqrt{ax+b}}{c} + \frac{bc-ad}{c}\int \frac{dx}{(cx+d)\,\sqrt{ax+b}}$$; (ver ejercicio 223)

238. $$\int \sqrt{(ax+b)\,(cx+d)}\,dx = \frac{2acx+bc+ad}{4ac}\sqrt{(ax+b)\,(cx+d)} - \frac{(bc-ad)^2}{8ac}\int \frac{dx}{(ax+b)\,(cx+d)}$$ (ver ejercicio 224)

239. $$\int \frac{dx}{(cx+d)\sqrt{(ax+b)\,(cx+d)}} = \frac{2\sqrt{ax+b}}{(ad-bc)\sqrt{cx+d}} + C$$

c. Cuando en las integrales figura la expresión ax^2+bx+c, ó la expresión $\sqrt{ax^2+bx+c}$

240. $$\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{4ac-b^2}}\operatorname{arc\,tg}\frac{2ax+b}{\sqrt{4ac-b^2}} + C;\ (4ac-b^2>0) =$$

$$= \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right| + C; \ (4ac - b^2 < 0)$$

241. $$\int \frac{dx}{\sqrt{ax^2 + bx + c}} = \ln \left| 2\sqrt{a(ax^2 + bx + c)} + 2ax + b \right| + C; \ (a > 0, 4ac - b^2 > 0) =$$

$$= \frac{1}{\sqrt{a}} \ln |2ax + b| + C; \ (a > 0, 4ac - b^2 = 0) =$$

$$= -\frac{1}{\sqrt{-a}} \operatorname{arc\,sen} \frac{2ax + b}{\sqrt{b^2 - 4ac}} + C; \ (a < 0, 4ac - b^2 < 0)$$

242. $$\int \frac{dx}{(ax^2 + bx + c)^2} = \frac{2ax + b}{(4ac - b^2)(ax^2 + bx + c)} + \frac{2a}{4ac - b^2} \int \frac{dx}{ax^2 + bx + c}; \ (\text{ver ejercicio } 240)$$

243. $$\int \frac{dx}{(ax^2 + bx + c)^3} = \frac{2ax + b}{4ac - b^2} \left(\frac{1}{2(ax^2 + bx + c)^2} + \frac{3a}{4ac - b^2 (ax^2 + bx + c)} \right) +$$

$$+ \frac{6a^2}{(4ac - b^2)^2} \int \frac{dx}{ax^2 + bx + c}; \ (\text{ver ejercicio } 240)$$

244. $$\int \frac{dx}{\sqrt{(ax^2 + bx + c)^3}} = \frac{2(2ax + b)}{(4ac - b^2)\sqrt{ax^2 + bx + c}} + C = \frac{2(bx + 2c)}{(b^2 - 4ac)\sqrt{ax^2 + bx + c}} + C$$

245. $$\int \frac{dx}{(ax^2 + bx + c)^n} = \frac{2ax + b}{(n - 1)(4ac - b^2)(ax^2 + bx + c)^{n-1}} +$$

$$+ \frac{(2n - 3)\,2a}{(n - 1)(4ac - b^2)} \int \frac{dx}{(ax^2 + bx + c)^{n-1}}$$

246. $$\int \frac{dx}{x(ax^2 + bx + c)} = \frac{1}{2c} \ln \left| \frac{x^2}{ax^2 + bx + c} \right| - \frac{b}{2c} \int \frac{dx}{ax^2 + bx + c}$$

247. $$\int \frac{dx}{x\sqrt{ax^2 + bx + c}} = -\frac{1}{\sqrt{c}} \ln \left| \frac{2\sqrt{c}\sqrt{ax^2 + bx + c} + bx + 2c}{x} \right| + C =$$

$$= \frac{1}{\sqrt{-c}} \operatorname{arc\,sen} \left| \frac{bx + 2c}{|x|\sqrt{b^2 - 4ac}} \right| + C$$

248. $$\int \frac{dx}{x^2(ax^2 + bx + c)} = \frac{b}{2c^2} \ln \left| \frac{ax^2 + bx + c}{x^2} \right| - \frac{1}{xc} + \frac{b^2 - 2ac}{2c^2} \int \frac{dx}{ax^2 + bx + c}$$

249. $$\int \frac{dx}{x^2\sqrt{ax^2+bx+c}} = -\frac{\sqrt{ax^2+bx+c}}{cx} - \frac{b}{2c}\int \frac{dx}{x\sqrt{ax^2+bx+c}}$$; (ver ejercicio 247)

250. $$\int \frac{dx}{x^n(ax^2+bx+c)} = -\frac{1}{(n-1)cx^{n-1}} - \frac{b}{c}\int \frac{dx}{x^{n-1}(ax^2+bx+c)} - \frac{a}{c}\int \frac{dx}{x^{n-2}(ax^2+bx+c)}$$

251. $$\int \frac{dx}{x(ax^2+bx+c)^2} = \frac{1}{2c(ax^2+bx+c)} - \frac{b}{2c}\int \frac{dx}{(ax^2+bx+c)^2} + \frac{1}{c}\int \frac{dx}{x(ax^2+bx+c)}$$; (ver ejercicio 246)

252. $$\int \frac{dx}{x\sqrt{(ax^2+bx+c)^3}} = \frac{1}{c\sqrt{ax^2+bx+c}} + \frac{1}{c}\frac{dx}{x\sqrt{ax^2+bx+c}} - \frac{b}{2c}\int \frac{dx}{\sqrt{(ax^2+bx+c)^3}}$$; (ver ejercicio 244)

253. $$\int \frac{dx}{x^2(ax^2+bx+c)^2} = \frac{-1}{cx(ax^2+bx+c)} - \frac{3a}{c}\int \frac{dx}{(ax^2+bx+c)^2} - \frac{2b}{c}\int \frac{dx}{x(ax^2+bx+c)^2}$$; (ver ejercicios 242 y 251)

254. $$\int \frac{dx}{x^2\sqrt{(ax^2+bx+c)^3}} = -\frac{ax^2+bx+c}{c^2x\sqrt{ax^2+bx+c}} - \frac{3b}{2c^2}\int \frac{dx}{x\sqrt{ax^2+bx+c}} + \frac{b^2-4ac}{2c^2}\int \frac{dx}{\sqrt{(ax^2+bx+c)^3}}$$; (ver ejercicios 247 y 244)

255. $$\int \frac{dx}{x(ax^2+bx+c)^n} = \frac{1}{2c(n-1)(ax^2+bx+c)^{n-1}} - \frac{b}{2c}\int \frac{dx}{(ax^2+bx+c)^n} + \frac{1}{c}\int \frac{dx}{x(ax^2+bx+c)^{n-1}}$$

256. $$\int \frac{dx}{x^m(ax^2+bx+c)^n} = \frac{-1}{(m-1)cx^{m-1}(ax^2+bx+c)^{n-1}} - \frac{(m+2n-3)a}{(m-1)c}\int \frac{dx}{x^{m-2}(ax^2+bx+c)^n} - \frac{(m+n-2)b}{(m-1)c}\int \frac{dx}{x^{m-1}(ax^2+bx+c)}$$; $(m \neq 1)$

257. $\int \frac{x\ dx}{ax^2+bx+c} = \frac{1}{2a}\ln|(ax^2+bx+c)| - \frac{b}{2a}\int \frac{dx}{ax^2+bx+c}$; (ver ejercicio 240)

258. $\int \frac{x\ dx}{\sqrt{ax^2+bx+c}} = \frac{\sqrt{ax^2+bx+c}}{a} - \frac{b}{2a}\int \frac{dx}{\sqrt{ax^2+bx+c}}$; (ver ejercicio 241)

259. $\int \frac{x\ dx}{(ax^2+bx+c)^2} = -\frac{bx+2c}{(4ac-b^2)\ (ax^2+bx+c)} - \frac{b}{4ac-b^2}\int \frac{dx}{ax^2+bx+c}$; (ver ejercicio 240)

260. $\int \frac{x\ dx}{(ax^2+bx+c)\sqrt{ax^2+bx+c}} = -\frac{2(bx+2c)}{(4ac-b^2)\sqrt{ax^2+bx+c}} + C$

261. $\int \frac{x\ dx}{(ax^2+bx+c)^n} = -\frac{bx+2c}{(n-1)\ (4ac-b^2)\ (ax^2+bx+c)^{n-1}} -$

$$-\frac{b(2n-3)}{(n-1)\ (4ac-b^2)}\int \frac{dx}{(ax^2+bx+c)^{n-1}}$$

262. $\int \frac{x^2dx}{ax^2+bx+c} = \frac{x}{a} - \frac{b}{2a^2}\ln|ax^2+bx+c| + \frac{b^2-2ac}{2a^2}\int \frac{dx}{ax^2+bx+c}$; (ver ejercicio 240)

263. $\int \frac{x^2dx}{\sqrt{ax^2+bx+c}} = \frac{2ax-3b}{4a^2}\sqrt{ax^2+bx+c} + \frac{3b^2-4ac}{8a^2}\int \frac{dx}{\sqrt{ax^2+bx+c}}$; (ver ejercicio 241)

264. $\int \frac{x^2dx}{x\sqrt{ax^2+bx+c}} = \frac{(2b^2-4ac)x+2bc}{a(4ac-b^2)\sqrt{ax^2+bx+c}} + \frac{1}{a}\int \frac{dx}{\sqrt{ax^2+bx+c}}$; (ver ejercicio 241)

265. $\int \frac{x^2dx}{(ax^2+bx+c)^2} = \frac{(b^2-2ac)x+bc}{a(4ac-b^2)\ (ax^2+bx+c)} + \frac{2c}{4ac-b^2}\int \frac{dx}{ax^2+bx+c}$

266. $\int \frac{x^2dx}{(ax^2+bx+c)^n} = -\frac{x}{(2n-3)\ (a)\ (ax^2+bx+c)^{n-1}} + \frac{c}{(2n-3)\ a}\int \frac{dx}{(ax^2+bx+c)^n} -$

$$-\frac{(n-2)\ b}{(2n-3)\ a}\int \frac{x\ dx}{(ax^2+bx+c)^n}$$; (ver ejercicios 245 y 261)

267. $\int \frac{x^n dx}{ax^2+bx+c} = \frac{x^{n-1}}{(n-1)\ a} - \frac{c}{a}\int \frac{x^{n-2}dx}{ax^2+bx+c} - \frac{b}{a}\int \frac{x^{n-1}dx}{ax^2+bx+c}$; $(n \neq 1)$

268. $\int \frac{x^m dx}{(ax^2+bx+c)^n} = -\frac{x^{m-1}}{(2n-m-1)\ a\ (ax^2+bx+c)^{n-1}} + \frac{(m-1)\ c}{(2n-m-1)\ a}\int \frac{x^{m-2}dx}{(ax^2+bx+c)} -$

$$-\frac{(n-m)\,b}{(2n-m-1)\,a}\int\frac{(ax^2+bx+c)^{m-1}dx}{(ax^2+bx+c)^n};\ (m\neq 2n-1),$$ si $m = 2n-1$ resulta:

269. $$\int\frac{x^{2n-1}dx}{(ax^2+bx+c)^n} = \frac{1}{a}\int\frac{x^{2n-3}dx}{(ax^2+bx+c)^{n-1}} - \frac{c}{a}\int\frac{x^{2n-3}dx}{(ax^2+bx+c)^n} - \frac{b}{a}\int\frac{x^{2n-2}dx}{(ax^2+bx+c)^n}$$

270. $$\int\sqrt{ax^2+bx+c}\,dx = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c}+\frac{4ac-b^2}{8a}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$

271. $$\int x\sqrt{ax^2+bx+c}\,dx = \frac{\sqrt{(ax^2+bx+c)^3}}{3a}-\frac{b\,(2ax+b)}{8a^2}\sqrt{ax^2+bx+c}-$$

$$-\frac{b\,(4ac-b^2)}{16a^2}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$

272. $$\int x^2\sqrt{ax^2+bx+c}\,dx = \frac{8ax-5b}{24a^2}\sqrt{(ax^2+bx+c)^3}+\frac{5b^2-4ac}{16a^2}\int\sqrt{ax^2+bx+c}\,dx$$

273. $$\int\frac{\sqrt{ax^2+bx+c}}{x}dx = \sqrt{ax^2+bx+c}+\frac{b}{2}\int\frac{dx}{\sqrt{ax^2+bx+c}}+c\int\frac{dx}{x\sqrt{ax^2+bx+c}}$$

274. $$\int\frac{x\ dx}{\sqrt{(ax^2+bx+c)^3}} = \frac{2\,(2ax+b)}{(4ac-b^2)\,\sqrt{ax^2+bx+c}}+C$$

275. $$\int\frac{x^2dx}{\sqrt{(ax^2+bx+c)^3}} = \frac{(2b^2-4ac)\,x+2bc}{a\,(4ac-b^2)\,\sqrt{ax^2+bx+c}}+\frac{1}{a}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$

276. $$\int\sqrt{(ax^2+bx+c)^{n+1}}\,dx = \frac{(2ax+b)\sqrt{(ax^2+bx+c)^{n+1}}}{4a\,(n+1)}+$$

$$+\frac{(2n+1)\ (4ac-b^2)}{8a\,(n+1)}\int\sqrt{(ax^2+bx+c)^{n-1}}\,dx$$

277. $$\int x\sqrt{(ax^2+bx+c)^{n+1}}\,dx = \frac{\sqrt{(ax^2+bx+c)^{n+3}}}{a\,(2n+3)}-\frac{b}{2a}\int\sqrt{(ax^2+bx+c)^{n+1}}\,dx$$

d. Cuando en las integrales figura la expresión $x^2 + a^2$, ó la expresión $\sqrt{x^2 + a^2}$

278. $\displaystyle\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\operatorname{arc\,tg}\frac{x}{a} + C$

279. $\displaystyle\int \frac{dx}{\sqrt{x^2 + a^2}} = \operatorname{arg\,sh}\frac{x}{a} + C = \ln\left|x + \sqrt{x^2 + a^2}\right| + C$

280. $\displaystyle\int \frac{dx}{(x^2 + a^2)^2} = \frac{x}{2a^2(x^2 + a^2)} + \frac{1}{2a^3}\operatorname{arc\,tg}\frac{x}{a} + C$

281. $\displaystyle\int \sqrt{x^2 + a^2}\,dx = \frac{x\sqrt{x^2 + a^2}}{2} + \frac{a^3}{2}\ln\left|x + \sqrt{x^2 + a^2}\right| + C$

282. $\displaystyle\int \frac{dx}{x(x^2 + a^2)} = \frac{1}{2a^2}\ln\left|\frac{x^2}{x^2 + a^2}\right| + C$

283. $\displaystyle\int \frac{dx}{x\sqrt{x^2 + a^2}} = -\frac{1}{2}\ln\left|\frac{a + \sqrt{x^2 + a^2}}{x}\right| + C$

284. $\displaystyle\int \frac{dx}{x^2(x^2 + a^2)} = -\frac{1}{a^2 x} - \frac{1}{a^3}\operatorname{arc\,tg}\frac{x}{a} + C$

285. $\displaystyle\int \frac{dx}{x^2\sqrt{x^2 + a^2}} = -\frac{\sqrt{x^2 + a^2}}{a^2 x} + C$

286. $\displaystyle\int \frac{dx}{x^3(x^2 + a^2)} = -\frac{1}{2a^2x^2} - \frac{1}{2a^4}\ln\left|\frac{x^2}{x^2 + a^2}\right| + C$

287. $\displaystyle\int \frac{dx}{x^3\sqrt{x^2 + a^2}} = -\frac{\sqrt{x^2 + a^2}}{2a^2x^2} + \frac{1}{2a^3}\ln\left|\frac{a + \sqrt{x^2 + a^2}}{x}\right| + C$

288. $\displaystyle\int \frac{x\,dx}{x^2 + a^2} = \frac{1}{2}\ln\left|(x^2 + a^2)\right| + C$

289. $\displaystyle\int x\sqrt{x^2 + a^2}\,dx = \frac{\sqrt{(x^2 + a^2)^3}}{3} + C$

290. $\displaystyle\int \frac{x^2\,dx}{x^2 + a^2} = x - a\operatorname{arc\,tg}\frac{x}{a} + C$

291. $\int x^2\sqrt{x^2+a^2}dx = \frac{x\sqrt{(x^2+a^2)^3}}{4} - \frac{a^2x\sqrt{x^2+a^2}}{8} - \frac{a^4}{8}\ln\left|x+\sqrt{x^2+a^2}\right| + C$

292. $\int \frac{x^3dx}{x^2+a^2} = \frac{x^2}{2} - \frac{a^2}{2}\ln\left|(x^2+a^2)\right| + C$

293. $\int x^3\sqrt{x^2+a^2}dx = \frac{\sqrt{(x^2+a^2)^5}}{5} - \frac{a^2\sqrt{(x^2+a^2)^3}}{3} + C$

294. $\int \frac{\sqrt{x^2+a^2}}{x}dx = \sqrt{x^2+a^2} - a\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right| + C$

295. $\int \frac{\sqrt{x^2+a^2}}{x^2}dx = -\frac{\sqrt{x^2+a^2}}{x} + \ln\left|x+\sqrt{x^2+a^2}\right| + C$

296. $\int \frac{\sqrt{x^2+a^2}}{x^3}dx = -\frac{\sqrt{x^2+a^2}}{2x^2} - \frac{1}{2a}\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right| + C$

297. $\int \frac{dx}{\sqrt{(x^2+a^2)^3}} = \frac{x}{a^2\sqrt{x^2+a^2}} + C$

298. $\int \frac{x\ dx}{(x^2+a^2)^2} = -\frac{1}{2(x^2+a^2)} + C$

299. $\int \frac{x\ dx}{\sqrt{(x^2+a^2)^3}} = -\frac{1}{\sqrt{x^2+a^2}} + C$

300. $\int \frac{x^2dx}{(x^2+a^2)^2} = -\frac{x}{2(x^2+a^2)} + \frac{1}{2a}\operatorname{arc\,tg}\frac{x}{a} + C$

301. $\int \frac{x\ dx}{\sqrt{(x^2+a^2)^3}} = -\frac{x}{\sqrt{x^2+a^2}} + \ln\left|x+\sqrt{x^2+a^2}\right| + C$

302. $\int \frac{x^3dx}{(x^2+a^2)^2} = \frac{a^2}{2(x^2+a^2)} + \frac{1}{2}\ln\left|(x^2+a^2)\right| + C$

303. $\int \frac{x^3dx}{\sqrt{(x^2+a^2)^3}} = \frac{1}{a^2\sqrt{x^2+a^2}} - \frac{1}{a^3}\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right| + C$

304. $\int \frac{dx}{x(x^2+a^2)^2} = \frac{1}{2a^2(x^2+a^2)} + \frac{1}{2a^4}\ln\left|\frac{x^2}{x^2+a^2}\right| + C$

305. $$\int\frac{dx}{x\sqrt{x^2+a^2}} = \frac{1}{a^2\sqrt{x^2+a^2}} - \frac{1}{a^3}\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right| + C$$

306. $$\int\frac{dx}{x^2(x^2+a^2)^2} = -\frac{1}{a^4x} - \frac{x}{2a^4(x^2+a^2)} - \frac{3}{2a^5}\operatorname{arc\,tg}\frac{x}{a} + C$$

307. $$\int\frac{dx}{x^2\sqrt{(x^2+a^2)^3}} = -\frac{\sqrt{x^2+a^2}}{a^4x} - \frac{x}{a^4\sqrt{x^2+a^2}} + C$$

308. $$\int\frac{dx}{x^3(x^2+a^2)^2} = -\frac{1}{2a^4(x^2+a^2)} - \frac{1}{a^5}\ln\left|\frac{x^2}{x^2+a^2}\right| + C$$

309. $$\int\frac{dx}{x^3\sqrt{(x^2+a^2)^3}} = -\frac{1}{2a^2x^2\sqrt{x^2+a^2}} - \frac{3}{2a^4\sqrt{x^2+a^2}} + \frac{3}{2a^5}\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right| + C$$

310. $$\int\frac{dx}{(x^2+a^2)^n} = \frac{x}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2}\int\frac{dx}{(x^2+a^2)^{n-1}}$$

311. $$\int\frac{x\ dx}{(x^2+a^2)^n} = -\frac{1}{2(n-1)\ (x^2+a^2)^{n-1}}$$

312. $$\int\frac{dx}{x(x^2+a^2)^n} = \frac{1}{2(n-1)\ (x^2+a^2)^{n-1}} + \frac{1}{2a}\int\frac{dx}{x(x^2+a^2)^{n-1}}$$

313. $$\int\sqrt{(x^2+a^2)^3}\,dx = \frac{x\sqrt{(x^2+a^2)^3}}{4} + \frac{3a^2x\sqrt{x^2+a^2}}{8} + \frac{3a^4}{8}\ln\left|x+\sqrt{x^2+a^2}\right| + C$$

314. $$\int x\sqrt{(x^2+a^2)^3}\,dx = \frac{\sqrt{(x^2+a^2)^5}}{5} + C$$

315. $$\int x^2\sqrt{(x^2+a^2)^3}\,dx = \frac{x\sqrt{(x^2+a^2)^5}}{6} - \frac{a^2x\sqrt{(x^2+a^2)^3}}{24} - \frac{a^4x\sqrt{x^2+a^2}}{16} - \frac{a^6}{16}\ln\left|x+\sqrt{x^2+a^2}\right| + C$$

316. $$\int x^3\sqrt{(x^2+a^2)^3}\,dx = \frac{\sqrt{(x^2+a^2)^7}}{7} - \frac{a^2\sqrt{(x^2+a^2)^5}}{5} + C$$

317. $$\int\frac{x^m dx}{(x^2+a^2)^n} = \int\frac{x^{m-2}dx}{(x^2+a^2)^{n-1}} - a^2\int\frac{x^{m-2}dx}{(x^2+a^2)^n}$$

318. $\int \frac{dx}{x^m (x^2+a^2)^n} = \frac{1}{a^2}\int \frac{dx}{x^m (x^2+a^2)^{n-1}} - \frac{1}{a^2}\int \frac{dx}{x^{m-2} (x^2+a^2)^n}$

319. $\int \frac{\sqrt{(x^2+a^2)^3}}{x} dx = \frac{\sqrt{(x^2+a^2)^3}}{3} + a^2\sqrt{x^2+a^2} - a^3 \ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right| + C$

320. $\int \frac{\sqrt{(x^2+a^2)^3}}{x^2} dx = -\frac{\sqrt{(x^2+a^2)^3}}{x} + \frac{3x\sqrt{x^2+a^2}}{2} + \frac{3}{2}a^2 \ln\left|x+\sqrt{x^2+a^2}\right| + C$

321. $\int \frac{\sqrt{(x^2+a^2)^3}}{x^3} dx = -\frac{\sqrt{(x^2+a^2)^3}}{2x} + \frac{3\sqrt{x^2+a^2}}{2} - \frac{3}{2}a \ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right| + C$

e. Cuando en las integrales figura la expresión $x^2 - a^2$,(siendo $x^2 > a^2$), ó la expresión $\sqrt{x^2 - a^2}$

322. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

323. $\int \frac{dx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2} + C$

324. $\int \frac{dx}{(x^2-a^2)^2} = -\frac{x}{2a^2(x^2-a^2)} - \frac{1}{4a^3}\ln\left|\frac{x-a}{x+a}\right| + C$

325. $\int \sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

326. $\int \frac{dx}{x(x^2-a^2)} = \frac{1}{2a^2}\ln\left|\frac{x^2-a^2}{x^2}\right| + C$

327. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\operatorname{arc\,sec}\left|\frac{x}{a}\right| + C$

328. $\int \frac{dx}{x^2(x^2-a^2)} = \frac{1}{a^2 x} + \frac{1}{2a^3}\ln\left|\frac{x-a}{x+a}\right| + C$

329. $\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2 x} + C$

330. $\int \frac{dx}{x^3(x^2-a^2)} = \frac{1}{2a^2x^2} - \frac{1}{2a^4}\ln\left|\frac{x^2}{x^2-a^2}\right| + C$

331. $$\int \frac{dx}{x^3\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{2a^2x^2} + \frac{1}{2a^3}\operatorname{arc\,sec}\left|\frac{x}{a}\right| + C$$

332. $$\int \frac{x\ dx}{x^2-a^2} = \frac{1}{2}\ln|x^2-a^2| + C$$

333. $$\int \frac{x\ dx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2} + C$$

334. $$\int \frac{x^2dx}{x^2-a^2} = x + \frac{a}{2}\ln\left|\frac{x-a}{x+a}\right| + C$$

335. $$\int \frac{x^2dx}{\sqrt{x^2-a^2}} = \frac{x\sqrt{x^2-a^2}}{2} + \ln\left|x+\sqrt{x^2-a^2}\right| + C$$

336. $$\int \frac{x^3dx}{x^2-a^2} = \frac{x^2}{2} + \frac{a^2}{2}\ln|x^2-a^2| + C$$

337. $$\int \frac{x^3dx}{\sqrt{x^2-a^2}} = \frac{\sqrt{(x^2-a^2)^3}}{3} + a^2\sqrt{x^2-a^2} + C$$

338. $$\int x^2\sqrt{x^2-a^2}dx = \frac{x\sqrt{(x^2-a^2)^3}}{4} + \frac{a^2x\sqrt{x^2-a^2}}{8} - \frac{a^4}{8}\ln\left|x+\sqrt{x^2-a^2}\right| + C$$

339. $$\int x^3\sqrt{x^2-a^2}dx = \frac{\sqrt{(x^2-a^2)^5}}{5} + \frac{a^2\sqrt{(x^2-a^2)^3}}{3} + C$$

340. $$\int \frac{\sqrt{x^2-a^2}}{x}dx = \sqrt{x^2-a^2} - a\ \operatorname{arc\,sec}\left|\frac{x}{a}\right| + C$$

341. $$\int \frac{\sqrt{x^2-a^2}}{x^2}dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln\left|x+\sqrt{x^2-a^2}\right| + C$$

342. $$\int \frac{\sqrt{x^2-a^2}}{x^3}dx = \frac{\sqrt{x^2-a^2}}{2x^2} + \frac{1}{2a}\operatorname{arc\,sec}\left|\frac{x}{a}\right| + C$$

343. $$\int \frac{dx}{\sqrt{(x^2-a^2)^3}} = -\frac{x}{a^2\sqrt{x^2-a^2}} + C$$

344. $$\int \frac{x\ dx}{(x^2-a^2)^2} = -\frac{1}{2(x^2-a^2)} + C$$

345. $\int \frac{x\ dx}{\sqrt{(x^2-a^2)^3}} = -\frac{1}{\sqrt{x^2-a^2}} + C$

346. $\int \frac{x^2 dx}{(x^2-a^2)^2} = -\frac{x}{2(x^2-a^2)} + \frac{1}{4a}\ln\left|\frac{x-a}{x+a}\right| + C$

347. $\int \frac{x^3 dx}{(x^2-a^2)^2} = -\frac{a^2}{2(x^2-a^2)} + \frac{1}{2}\ln|x^2-a^2| + C$

348. $\int \frac{x^2 dx}{\sqrt{(x^2-a^2)^3}} = -\frac{x}{\sqrt{x^2-a^2}} + \ln\left|x+\sqrt{x^2-a^2}\right| + C$

349. $\int \frac{x^3 dx}{\sqrt{(x^2-a^2)^3}} = \sqrt{x^2-a^2} - \frac{a^2}{\sqrt{x^2-a^2}} + C$

350. $\int \frac{dx}{x(x^2-a^2)^2} = -\frac{1}{2a^2(x^2-a^2)} + \frac{1}{2a^4}\ln\left|\frac{x^2}{x^2-a^2}\right| + C$

351. $\int \frac{dx}{x^2(x^2-a^2)^2} = -\frac{1}{a^4 x} - \frac{x}{2a^4(x^2-a^2)} - \frac{3}{4a^5}\ln\left|\frac{x-a}{x+a}\right| + C$

352. $\int \frac{dx}{x\sqrt{(x^2-a^2)^3}} = -\frac{1}{a^2\sqrt{x^2-a^2}} - \frac{1}{a^3}\operatorname{arc\,sec}\left|\frac{x}{a}\right| + C$

353. $\int \frac{dx}{x^2\sqrt{(x^2-a^2)^3}} = -\frac{\sqrt{x^2-a^2}}{a^{4x}} - \frac{x}{a^4\sqrt{x^2-a^2}} + C$

354. $\int \frac{dx}{x^3(x^2-a^2)^2} = -\frac{1}{2a^4x^2} - \frac{1}{2a^4(x^2-a^2)} + \frac{1}{a^5}\ln\left|\frac{x^2}{x^2-a^2}\right| + C$

355. $\int \frac{dx}{x^3\sqrt{(x^2-a^2)^3}} = \frac{1}{2a^2x^2\sqrt{x^2-a^2}} - \frac{3}{2a^4\sqrt{x^2-a^2}} - \frac{3}{2a^5}\operatorname{arc\,sec}\left|\frac{x}{a}\right| + C$

356. $\int \sqrt{(x^2-a^2)^3}\,dx = \frac{x\sqrt{(x^2-a^2)^3}}{4} - \frac{3a^2x\sqrt{x^2-a^2}}{8} + \frac{3a^4}{8}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

357. $\int x\sqrt{(x^2-a^2)^3}\,dx = \frac{\sqrt{(x^2-a^2)^5}}{5} + C$

358. $\int x^2\sqrt{(x^2-a^2)^3}\,dx = \frac{x\sqrt{(x^2-a^2)^5}}{6} + \frac{a^2x\sqrt{(x^2-a^2)^3}}{24} -$

$$-\frac{a^4x\sqrt{x^2-a^2}}{16}+\frac{a^6}{16}\ln\left|x+\sqrt{x^2-a^2}\right|+C$$

359. $\displaystyle\int x^3\sqrt{(x^2-a^2)^3}\,dx = \frac{\sqrt{(x^2-a^2)^7}}{7}+\frac{a^2\sqrt{(x^2-a^2)^5}}{5}+C$

360. $\displaystyle\int \frac{\sqrt{(x^2-a^2)^3}}{x}\,dx = \frac{\sqrt{(x^2-a^2)^3}}{3}-a^2\sqrt{x^2-a^2}+a^3\operatorname{arc\,sec}\left|\frac{x}{a}\right|+C$

361. $\displaystyle\int \frac{\sqrt{(x^2-a^2)^3}}{x^2}\,dx = -\frac{\sqrt{(x^2-a^2)^3}}{x}+\frac{3x\sqrt{x^2-a^2}}{2}-\frac{3}{a^2}\ln\left|x+\sqrt{x^2-a^2}\right|+C$

362. $\displaystyle\int \frac{\sqrt{(x^2-a^2)^3}}{x^3}\,dx = -\frac{\sqrt{(x^2-a^2)^3}}{2x^2}+\frac{3\sqrt{x^2-a^2}}{2}-\frac{3}{2}a\operatorname{arc\,sec}\left|\frac{x}{a}\right|+C$

363. $\displaystyle\int \frac{dx}{(x^2-a^2)^n} = \frac{-x}{2(n-1)a^2(x^2-a^2)^{n-1}}-\frac{2n-3}{(2n-2)a^2}\int\frac{dx}{(x^2-a^2)^{n-1}}$

364. $\displaystyle\int \frac{x\ dx}{(x^2-a^2)^n} = -\frac{1}{2(n-1)\ (x^2-a^2)^{n-1}}+C$

365. $\displaystyle\int \frac{dx}{x(x^2-a^2)^n} = -\frac{1}{2(n-1)a^2(x^2-a^2)^{n-1}}-\frac{1}{a^2}\int\frac{dx}{x(x^2-a^2)^{n-1}}$

366. $\displaystyle\int \frac{x^m dx}{(x^2-a^2)^n} = \int\frac{x^{m-2}dx}{x^2-a^{2n-1}}+a^2\int\frac{x^{m-2}dx}{(x^2-a^2)^n}$

367. $\displaystyle\int \frac{dx}{x^m(x^2-a^2)^n} = \frac{1}{a^2}\int\frac{dx}{x^{m-2}(x^2-a^2)^n}-\frac{1}{a^2}\int\frac{dx}{x^m(x^2-a^2)^{n-1}}$

f. Cuando en las integrales figura la expresión a^2-x^2, (siendo $x^2<a^2$), ó la expresión $\sqrt{a^2-x^2}$

368. $\displaystyle\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+C$

369. $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{arc\,sen}\frac{x}{a}+C$

370. $\displaystyle\int \frac{dx}{(a^2-x^2)^2} = \frac{x}{2a^2(a^2-x^2)}+\frac{1}{4a^3}\ln\left|\frac{a+x}{a-x}\right|+C$

371. $\int \sqrt{a^2 - x^2}\,dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \operatorname{arc\,sen} \frac{x}{a} + C$

372. $\int \frac{dx}{x\,(a^2 - x^2)} = \frac{1}{2a^2} \ln \left| \frac{x^2}{a^2 - x^2} \right| + C$

373. $\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$

374. $\int \frac{dx}{x^2\,(a^2 - x^2)} = -\frac{1}{a^2 x} + \frac{1}{2a^3} \ln \left| \frac{a + x}{a - x} \right| + C$

375. $\int \frac{dx}{x^2\sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C$

376. $\int \frac{dx}{x^3\,(a^2 - x^2)} = -\frac{1}{2a^2 x^2} + \frac{1}{2a^4} \ln \left| \frac{x^2}{a^2 - x^2} \right| + C$

377. $\int \frac{dx}{x^3\sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{2a^2 x^2} - \frac{1}{2a^3} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$

378. $\int \frac{x\,dx}{(a^2 - x^2)^2} = \frac{1}{2\,(a^2 - x^2)} + C$

379. $\int \frac{x\,dx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2} + C$

380. $\int \frac{x^2 dx}{a^2 - x^2} = -x + \frac{a}{2} \ln \left| \frac{a + x}{a - x} \right| + C$

381. $\int \frac{x^2 dx}{\sqrt{a^2 - x^2}} = -\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \operatorname{arc\,sen} \frac{x}{a} + C$

382. $\int \frac{x^3 dx}{a^2 - x^2} = -\frac{x^2}{2} - \frac{a^2}{2} \ln |a^2 - x^2| + C$

383. $\int \frac{x^3 dx}{\sqrt{a^2 - x^2}} = \frac{\sqrt{(a^2 - x^2)^3}}{3} - a^2\sqrt{a^2 - x^2} + C$

384. $\int x\sqrt{a^2 - x^2}\,dx = -\frac{\sqrt{(a^2 - x^2)^3}}{3} + C$

385. $\int \frac{x^2 dx}{(a^2 - x^2)^2} = \frac{x}{2(a^2 - x^2)} \ln\left|\frac{a + x}{a - x}\right| + C$

386. $\int x^2 \sqrt{a^2 - x^2} dx = -\frac{x\sqrt{(a^2 - x^2)^3}}{4} + \frac{a^2 x\sqrt{a^2 - x^2}}{8} - \frac{a^4}{8} \text{arc sen} \frac{x}{a} + C$

387. $\int \frac{x^3 dx}{(a^2 - x^2)^2} = \frac{a^2}{2(a^2 - x^2)} + \frac{1}{2} \ln|a^2 - x^2| + C$

388. $\int x^3 \sqrt{a^2 - x^2} dx = \frac{\sqrt{(a^2 - x^2)^5}}{5} - \frac{a^2\sqrt{(a^2 - x^2)^3}}{3} + C$

389. $\int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln\left|\frac{a + \sqrt{a^2 - x^2}}{x}\right| + C$

390. $\int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \text{arc sen} \frac{x}{a} + C$

391. $\int \frac{dx}{x(a^2 - x^2)^2} = \frac{1}{2a^2(a^2 - x^2)} + \frac{1}{2a^4} \ln\left|\frac{x^2}{a^2 - x^2}\right| + C$

392. $\int \frac{dx}{x^2(a^2 - x^2)^2} = -\frac{1}{a^4 x} + \frac{x}{2a^4(a^2 - x^2)} + \frac{3}{4a^5} \ln\left|\frac{a + x}{a - x}\right| + C$

393. $\int \frac{dx}{x^3(a^2 - x^2)^2} = -\frac{1}{2a^4 x^2} + \frac{1}{2a^4(a^2 - x^2)} + \frac{1}{a^6} \ln\left|\frac{x^2}{a^2 - x^2}\right| + C$

394. $\int \frac{\sqrt{a^2 - x^2}}{x^3} dx = -\frac{\sqrt{a^2 - x^2}}{2x^2} + \frac{1}{2a} \ln\left|\frac{a + \sqrt{a^2 - x^2}}{x}\right| + C$

395. $\int \frac{dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{a^2\sqrt{a^2 - x^2}} + C$

396. $\int \frac{x\ dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{\sqrt{a^2 - x^2}} + C$

397. $\int \frac{x^2 dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{\sqrt{a^2 - x^2}} - \text{arc sen} \frac{x}{a} + C$

398. $\int \frac{x^3 dx}{\sqrt{(a^2 - x^2)^3}} = \sqrt{a^2 - x^2} + \frac{a^2}{\sqrt{a^2 - x^2}} + C$

399. $\int \sqrt{(a^2-x^2)^3}\,dx = \frac{x\sqrt{(a^2-x^2)^3}}{4} + \frac{3a^2x\sqrt{a^2-x^2}}{8} + \frac{3a^4}{8}\text{arc sen}\frac{x}{a} + C$

400. $\int x\sqrt{(a^2-x^2)^3}\,dx = -\frac{\sqrt{(a^2-x^2)^5}}{5} + C$

401. $\int x^2\sqrt{(a^2-x^2)}\,dx = -\frac{x\sqrt{(a^2-x^2)^5}}{6} + a^2x\sqrt{(a^2-x^2)^3} + \frac{a^4x\sqrt{a^2-x^2}}{16} + \frac{a^6}{16}\text{arc sen}\frac{x}{a} + C$

402. $\int x^3\sqrt{(a^2-x^2)^3}\,dx = \frac{\sqrt{(a^2-x^2)^7}}{7} - \frac{a^2\sqrt{(a^2-x^2)^5}}{5} + C$

403. $\int \frac{\sqrt{(a^2-x^2)^3}}{x}\,dx = \frac{\sqrt{(a^2-x^2)^3}}{3} + a^2\sqrt{a^2-x^2} - a^3\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$

404. $\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\frac{\sqrt{(a^2-x^2)^3}}{x} - \frac{3x\sqrt{a^2-x^2}}{2} - \frac{3}{2}a^2\text{arc sen}\frac{x}{a} + C$

405. $\int \frac{\sqrt{a^2-x^2}}{x^3}\,dx = -\frac{\sqrt{(a^2-x^2)^3}}{2x^2} - \frac{3\sqrt{a^2-x^2}}{2} + \frac{3}{2}a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$

406. $\int \frac{dx}{x\sqrt{(a^2-x^2)^3}} = \frac{1}{a^2\sqrt{a^2-x^2}} - \frac{1}{a^3}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$

407. $\int \frac{dx}{x^2\sqrt{(a^2-x^2)^3}} = -\frac{\sqrt{a^2-x^2}}{a^4x} + \frac{x}{a^4\sqrt{a^2-x^2}} + C$

408. $\int \frac{dx}{x^3\sqrt{(a^2-x^2)^3}} = -\frac{1}{2a^2x^2\sqrt{a^2-x^2}} + \frac{3}{2a^4\sqrt{a^2-x^2}} - \frac{3}{2a^5}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$

409. $\int \frac{dx}{(a^2-x^2)^n} = \frac{x}{2(n-1)a^2(a^2-x^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2}\int \frac{dx}{(a^2-x^2)^{n-1}}$

410. $\int \frac{x\ dx}{(a^2-x^2)^n} = \frac{1}{2(n-1)\ (a^2-x^2)^{n-1}} + C$

411. $\int \frac{x^m dx}{(a^2-x^2)^n} = a^2\int \frac{x^{m-2}dx}{(a^2-x^2)^n} - \int \frac{x^{m-2}dx}{(a^2-x^2)^{n-1}}$

412. $\int \frac{dx}{x(a^2-x^2)^n} = \frac{1}{2(n-1)a^2(a^2-x^2)^{n-1}} + \frac{1}{a^2}\int \frac{dx}{x(a^2-x^2)^{n-1}}$

413. $\displaystyle\int\frac{dx}{x^m(a^2-x^2)^n} = \frac{1}{a^2}\int\frac{dx}{x^{m-2}(a^2-x^2)^n}+\frac{1}{a^2}\int\frac{dx}{x^m(a^2-x^2)^{n-1}}$

g. Cuando en las integrales figuran las expresiones $x^3 \pm a^3; x^4 \pm a^4$ y $x^n \pm a^n$

414. $\displaystyle\int\frac{dx}{x^3+a^3} = \frac{1}{6a^2}\ln\left|\frac{(x+a)^2}{x^2-ax+a^2}\right|+\frac{1}{a^2\sqrt{3}}\operatorname{arc\,tg}\frac{2x-a}{a\sqrt{3}}+C$

415. $\displaystyle\int\frac{dx}{x^3-a^3} = -\frac{1}{6a^2}\ln\left|\frac{(x-a)^2}{x^2+ax+a^2}\right|+\frac{1}{a^2\sqrt{3}}\operatorname{arc\,tg}\frac{2x+a}{a\sqrt{3}}+C$

416. $\displaystyle\int\frac{dx}{x^4+a^4} = \frac{1}{4a^3\sqrt{2}}\ln\left|\frac{x^2+ax\sqrt{2}+a^2}{x^2-ax\sqrt{2}+a^2}\right|-\frac{1}{2a^3\sqrt{2}}\operatorname{arc\,tg}\frac{ax\sqrt{2}}{x^2-a^2}+C$

417. $\displaystyle\int\frac{dx}{x^4-a^4} = \frac{1}{4a^3}\ln\left|\frac{x-a}{x+a}\right|-\frac{1}{2a^3}\operatorname{arc\,tg}\frac{x}{a}+C$

418. $\displaystyle\int\frac{dx}{x(x^3+a^3)} = \frac{1}{3a^3}\ln\left|\frac{x^3}{x^3+a^3}\right|+C$

419. $\displaystyle\int\frac{dx}{x(x^3-a^3)} = -\frac{1}{a^3}\ln\left|\frac{x^3}{x^3-a^3}\right|+C$

420. $\displaystyle\int\frac{dx}{x(x^4+a^4)} = \frac{1}{4a^4}\ln\left|\frac{x^4}{x^4+a^4}\right|+C$

421. $\displaystyle\int\frac{dx}{x(x^n+a^n)} = \frac{1}{na^n}\ln\left|\frac{x^n}{x^n+a^n}\right|+C$

422. $\displaystyle\int\frac{dx}{x(x^n-a^n)} = \frac{1}{na^n}\ln\left|\frac{x^n-a^n}{x^n}\right|+C$

423. $\displaystyle\int\frac{dx}{x^2(x^3+a^3)} = -\frac{1}{a^3x}-\frac{1}{6a^4}\ln\left|\frac{x^2-ax+a^2}{(x+a)^2}\right|-\frac{1}{a^4\sqrt{3}}\operatorname{arc\,tg}\frac{2x-a}{a\sqrt{3}}+C$

424. $\displaystyle\int\frac{dx}{x^2(x^4+a^4)} = -\frac{1}{a^4x}-\frac{1}{4a^5\sqrt{2}}\ln\left|\frac{x^2-ax\sqrt{2}+a^2}{x^2+ax\sqrt{2}+a^2}\right|+\frac{1}{2a^5\sqrt{2}}\operatorname{arc\,tg}\frac{ax\sqrt{2}}{x^2-a^2}+C$

425. $\displaystyle\int\frac{dx}{(x^3+a^3)^2} = \frac{x}{3a^3(x^3+a^3)}+\frac{1}{9a^5}\ln\left|\frac{(x+a)^2}{x^2-ax+a^2}\right|+\frac{2}{3a^5\sqrt{3}}\operatorname{arc\,tg}\frac{2x-a}{a\sqrt{3}}+C$

209. $\int x\sqrt{(ax+b)^3} = \frac{2}{35a^2}(5\sqrt{ax+b^7} - 7b\sqrt{ax+b^5}) + C$

210. $\int x^2\sqrt{(ax+b)^3} = \frac{2}{a^3}\left(\frac{\sqrt{(ax+b)^9}}{9} - \frac{2b\sqrt{(ax+b)^7}}{7} + \frac{b^2\sqrt{(ax+b)^5}}{5}\right) + C$

211. $\int x\sqrt{(ax+b)^n}dx = \frac{2\sqrt{(ax+b)^{n+4}}}{a^2(n+4)} - \frac{2b\sqrt{(ax+b)^{n+2}}}{a^2(n+2)} + C$

212. $\int x^2\sqrt{(ax+b)^n}dx = \frac{2\sqrt{(ax+b)^{n+6}}}{a^3(n+6)} - \frac{4b\sqrt{(ax+b)^{n+4}}}{a^3(n+4)} + \frac{2b^2\sqrt{(ax+b)^{n+2}}}{a^3(n+2)} + C$

213. $\int \frac{\sqrt{(ax+b)^3}}{x}dx = \frac{2\sqrt{(ax+b)^3}}{3} + 2b\sqrt{(ax+b)} + b^2\int\frac{dx}{x\sqrt{ax+b}}$

214. $\int \frac{x\ dx}{\sqrt{(ax+b)^3}} = \frac{2}{a^2}(\sqrt{ax+b} + \frac{b}{\sqrt{ax+b}}) + C$

215. $\int \frac{x^2dx}{\sqrt{(ax+b)^3}} = \frac{2}{a^3}\left(\frac{\sqrt{(ax+b)^3}}{3} - 2b\sqrt{ax+b} - \frac{b^2}{\sqrt{ax+b}}\right) + C$

216. $\int \frac{dx}{x\sqrt{(ax+b)^3}} = \frac{2}{b\sqrt{ax+b}} + \frac{1}{b}\int\frac{dx}{x\sqrt{ax+b}}$

217. $\int \frac{dx}{x^2\sqrt{(ax+b)^3}} = -\frac{1}{bx\sqrt{ax+b}} - \frac{3a}{b^2\sqrt{ax+b}} - \frac{3a}{2b^2}\int\frac{dx}{x\sqrt{ax+b}}$

b. Cuando en las integrales figura alguna de las siguientes expresiones:
b_1) $ax + b$ y $cx + d$; **b_2)** $\sqrt{ax+b}$ y $cx+d$; **b_3)** $\sqrt{ax+b}$ y $\sqrt{cx+d}$

218. $\int \frac{ax+b}{cx+d}dx = \frac{ax}{c} + \frac{bc-ad}{c^2}\ln|(cx+d)| + C$

219. $\int \frac{\sqrt{ax+b}}{cx+d}dx = \frac{2\sqrt{ax+b}}{c} - \frac{2\sqrt{ad-bc}}{c\sqrt{c}}\operatorname{arc\,tg}\sqrt{\frac{c(ax+b)}{ad-bc}} + C =$

$$= \frac{2\sqrt{ax+b}}{c} + \frac{\sqrt{bc-ad}}{c\sqrt{c}}\ln\left|\frac{\sqrt{c(ax+b)} - \sqrt{bc-ad}}{\sqrt{c(ax+b)} + \sqrt{bc-ad}}\right| + C;\ (bc-ad) \neq 0$$

220. $\int \frac{cx+d}{\sqrt{ax+b}}dx = \frac{2\,(acx+3ad-2bc)}{3a^2}\sqrt{ax+b}+C$

221. $\int \frac{\sqrt{cx+d}}{\sqrt{ax+b}}dx = \frac{\sqrt{(ax+b)\,(cx+d)}}{a} + \frac{(ad-bc)}{2a}\int \frac{dx}{\sqrt{(ax+b)\,(cx+d)}}$ (verejercicio224)

222. $\int \frac{dx}{(ax+b)\,(cx+d)} = \frac{1}{bc-ad}\ln\left|\frac{cx+d}{ax+b}\right| + C;\ (bc-ad\neq 0)$

223. $\int \frac{dx}{(cx+d)\,\sqrt{ax+b}} = \frac{2}{\sqrt{ad-bc}}\operatorname{arc\,tg}\sqrt{\frac{c\,(ax+b)}{ad-bc}} + C =$

$$= \frac{1}{\sqrt{bc-ad}\sqrt{c}}\ln\left|\frac{\sqrt{c\,(ax+b)}-\sqrt{bc-ad}}{\sqrt{c\,(ax+b)}-\sqrt{bc-ad}}\right| + C;\ (bc-ad\neq 0)$$

224. $\int \frac{dx}{\sqrt{(ax+b)\,(cx+d)}} = \frac{2}{\sqrt{-ac}}\operatorname{arc\,tg}\sqrt{\frac{-c\,(ax+b)}{a\,(cx+d)}} + C =$

$$= \frac{2}{\sqrt{ac}}\ln\left|\sqrt{a\,(cx+d)}+\sqrt{c\,(ax+b)}\right| + C$$

225. $\int \frac{dx}{(ax+b)^2\,(cx+d)} = \frac{1}{bc-ad}\left(\frac{1}{ax+b} + \frac{c}{bc-ad}\ln\left|\frac{cx+d}{ax+b}\right|\right) + C;\ (bc-ad\neq 0)$

226. $\int \frac{dx}{\sqrt{(ax+b)}\,\sqrt{(cx+d)^3}} = -\frac{2\sqrt{(ax+b)}}{(bc-ad)\,\sqrt{cx+d}} + C;\ (bc-ad\neq 0)$

227. $\int \frac{x\ dx}{(ax+b)\,(cx+d)} = \frac{1}{bc-ad}\left[\frac{b}{a}\ln|\,(ax+b)\,| - \frac{d}{c}\ln|\,(cx+d)\,|\right] + C;\ (bc-ad\neq 0)$

228. $\int \frac{x\ dx}{\sqrt{(ax+b)\,(cx+d)}} = \frac{\sqrt{(ax+b)\,(cx+d)}}{ac} - \frac{bc+ad}{2ac}\int \frac{dx}{\sqrt{(ax+b)\,(cx+d)}}$

229. $\int \frac{x\ dx}{(ax+b)^2\,(cx+d)} = \frac{1}{bc-ad}\left[\frac{d}{bc-ad}\ln\left|\frac{ax+b}{cx+d}\right| - \frac{b}{a\,(ax+b)}\right] + C;\ (bc-ad\neq 0)$

230. $\int \frac{x^2 dx}{(ax+b)^2\,(cx+d)} = \frac{b^2}{(bc-ad)\,a^2\,(ax+b)}$

$$+ \frac{1}{(bc-ad)^2}\left[\frac{d^2}{c}\ln|\,(cx+d)\,| + \frac{b\,(bc-ad)}{a^2}\ln|\,(ax+b)\,|\right] + C$$

231. $\int \frac{dx}{(ax+b)^m\,(cx+d)^n} = -\frac{1}{(n-1)\,(bc-ad)}\left[\frac{1}{(ax+b)^{m-1}\,(cx+d)^{n-1}} +\right.$

454. $$\int \frac{dx}{(a^2+b^2x)\sqrt{x^3}} = -\frac{2}{a^2\sqrt{x}} - \frac{2b}{a^3}\left[\frac{1}{2}\ln\left|\frac{a+b\sqrt{x}}{a-b\sqrt{x}}\right|\right] + C$$

455. $$\int \frac{dx}{(a^2-b^2x)\sqrt{x^3}} = -\frac{2}{a^2\sqrt{x}} + \frac{2b}{a^3}\left(\operatorname{arc\,tg}\frac{b\sqrt{x}}{a}\right) + C$$

456. $$\int \frac{dx}{(a^2+b^2x)^2\sqrt{x}} = \frac{\sqrt{x}}{a^2x} + \frac{1}{a^3b}\operatorname{arc\,tg}\frac{b\sqrt{x}}{a} + C$$

457. $$\int \frac{dx}{(a^2+b^2x)^2\sqrt{x^3}} = -\frac{2}{a^2(a^2+b^2x)\sqrt{x}} - \frac{3b^2\sqrt{x}}{a^4(a^2+b^2x)} - \frac{3b}{a^5}\left[\frac{2b}{a^3}\left(\operatorname{arc\,tg}\frac{b\sqrt{x}}{a}\right)\right] + C$$

458. $$\int \frac{dx}{(a^2-b^2x)^2\sqrt{x^3}} = -\frac{2}{a^2(a^2-b^2x)\sqrt{x}} + \frac{3b^2\sqrt{x}}{a^4(a^2-b^2x)}\operatorname{arc\,tg}\frac{b\sqrt{x}}{a} + C$$

459. $$\int \frac{\sqrt{x}dx}{a^2+b^2x} = \frac{2\sqrt{x}}{b^2} - \frac{2a}{b^3}\left(\operatorname{arc\,tg}\frac{b\sqrt{x}}{a}\right) + C$$

460. $$\int \frac{\sqrt{x}dx}{a^2-b^2x} = -\frac{2\sqrt{x}}{b^2} + \frac{2a}{b^3}\left[\frac{1}{2}\ln\left|\frac{a+b\sqrt{x}}{a-b\sqrt{x}}\right|\right] + C$$

461. $$\int \frac{\sqrt{x}dx}{(a^2+b^2x)^2} = \frac{\sqrt{x}}{b^2(a^2+b^2x)} + \frac{1}{ab^3}\left(\operatorname{arc\,tg}\frac{b\sqrt{x}}{a}\right) + C$$

462. $$\int \frac{\sqrt{x}dx}{(a^2-b^2x)^2} = -\frac{\sqrt{x}}{b^2(a^2-b^2x)} - \frac{1}{ab^3}\left[\frac{1}{2}\ln\left|\frac{a+b\sqrt{x}}{a-b\sqrt{x}}\right|\right] + C$$

463. $$\int \frac{\sqrt{x^3}dx}{a^2+b^2x} = \frac{2}{3}\frac{\sqrt{x^3}}{b^2} - \frac{2a^2\sqrt{x}}{b^4} + \frac{2a^3}{b^5}\left(\operatorname{arc\,tg}\frac{b\sqrt{x}}{a}\right) + C$$

464. $$\int \frac{\sqrt{x^3}dx}{a^2-b^2x} = -\frac{2\sqrt{x^3}}{3b^2} - \frac{2a^2\sqrt{x}}{b^4} + \frac{2a^3}{b^5}\left[\frac{1}{2}\ln\left|\frac{a+b\sqrt{x}}{a-b\sqrt{x}}\right|\right] + C$$

465. $$\int \frac{\sqrt{x^3}dx}{(a^2+b^2x)^2} = \frac{2\sqrt{x^3}}{b^2x} + \frac{3a^2\sqrt{x}}{b^4(a^2+b^2x)} - \frac{3a}{b^5}\left(\operatorname{arc\,tg}\frac{b\sqrt{x}}{a}\right) + C$$

466. $$\int \frac{\sqrt{x^3}dx}{(a^2-b^2x)^2} = -\frac{2\sqrt{x^3}}{b^2x} + \frac{3a^2\sqrt{x}}{b^4(a^2-b^2x)} - \frac{3a}{b^5}\left[\frac{1}{2}\ln\left|\frac{a+b\sqrt{x}}{a-b\sqrt{x}}\right|\right] + C$$

B$_4$. INTEGRALES DE FUNCIONES TRASCENDENTES

a. Integrales de funciones trigonométricas

a$_1$. Integrales en las que figura la función seno

467. $\int \operatorname{sen} ax \; dx = -\frac{\cos ax}{a} + C$

468. $\int \operatorname{sen}^2 ax \; dx = \frac{x}{2} - \frac{\operatorname{sen} 2ax}{4a} + C$

469. $\int \operatorname{sen}^3 ax \; dx = -\frac{\cos ax}{a} + \frac{\cos^3 ax}{3a} + C$

470. $\int \operatorname{sen}^4 ax \; dx = \frac{3x}{8} - \frac{\operatorname{sen} 2ax}{4a} + \frac{\operatorname{sen} 4ax}{32a} + C$

471. $\int \operatorname{sen}^n ax \; dx = -\frac{\operatorname{sen}^{n-1} ax \; \cos ax}{an} + \frac{n-1}{n}\int \operatorname{sen}^{n-2} ax \; dx$

472. $\int x \; \operatorname{sen} ax \; dx = \frac{\operatorname{sen} ax}{a^2} - \frac{x \; \cos ax}{a} + C$

473. $\int x \; \operatorname{sen}^2 ax \; dx = \frac{x^2}{4} - \frac{x \; \operatorname{sen} 2ax}{4a} - \frac{\cos 2ax}{8a^2} + C$

474. $\int x^2 \; \operatorname{sen} ax \; dx = \frac{2x}{a^2} \operatorname{sen} ax + \left(\frac{2}{a^3} - \frac{x^2}{a}\right) \cos ax + C$

475. $\int x^3 \; \operatorname{sen} ax \; dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4}\right) \operatorname{sen} ax + \left(\frac{6x}{a^3} - \frac{x^2}{a}\right) \cos ax + C$

476. $\int x^n \; \operatorname{sen} ax \; dx = -\frac{x^n}{a} \cos ax + \frac{n}{a}\int x^{n-1} \cos ax \; dx; \; (n > 0)$

477. $\int \frac{dx}{\operatorname{sen} ax} = \frac{1}{a} \ln \left(\operatorname{cosec} ax - \operatorname{cotg} ax\right) + C = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2} + C$

478. $\int \frac{dx}{\operatorname{sen}^2 ax} = -\frac{1}{a} \operatorname{cotg} ax + C$

479. $\int \frac{dx}{\operatorname{sen}^3 ax} = -\frac{\cos ax}{2a \; \operatorname{sen}^2 ax} + \frac{1}{2a} \ln \left| \operatorname{tg} \frac{ax}{2} \right| + C$

480. $\int \frac{dx}{\operatorname{sen}^n ax} = \frac{-\cos ax}{a\,(n-1) \operatorname{sen}^{n-1} ax} + \frac{n-2}{n-1}\int \frac{dx}{\operatorname{sen}^{n-2} ax}$

481. $\int \frac{\operatorname{sen} ax}{x} dx = ax - \frac{(ax)^3}{3.3!} + \frac{(ax)^5}{5.5!} - \frac{(ax)^7}{7.7!} + \dots\dots$

482. $\int \frac{\operatorname{sen} ax}{x^2} dx = -\frac{\operatorname{sen} ax}{x} + a\int \frac{\cos ax}{x} dx$ (ver 516)

483. $\int \frac{\operatorname{sen} ax}{x^n} dx = -\frac{\operatorname{sen} ax}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{\cos ax}{x^{n-1}} dx$ (ver 518)

484. $\int \frac{x\ dx}{\operatorname{sen} ax} = \frac{1}{a^2}\left[ax + \frac{(ax)^3}{3.3!} + \frac{7(ax)^5}{3.5.5!} + \frac{31(ax)^7}{3.7.7!} + \frac{27(ax)^9}{3.9.9!} + \dots + \right.$

$\left. + \frac{2(2^{2n-1}-1)Bn(ax)^{2n+1}}{(2n+1)!} + \dots\right]$ (siendo Bn: número de Bernoulli)

485. $\int \frac{x\ dx}{\operatorname{sen}^n ax} = \frac{-x\ \cos ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \frac{1}{a^2(n-1)\ (n-2)\operatorname{sen}^{n-2} ax} + \frac{n-2}{n-1}\int \frac{x\ dx}{\operatorname{sen}^{n-2} ax}$; $(n > 2)$

486. $\int \frac{dx}{1 + \operatorname{sen} ax} = -\frac{1}{a}\operatorname{tg}\left(\frac{\pi}{4} - \frac{ax}{2}\right) + C$

487. $\int \frac{dx}{1 - \operatorname{sen} ax} = \frac{1}{a}\operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right) + C$

488. $\int \frac{dx}{(1 + \operatorname{sen} ax)^2} = -\frac{1}{2a}\operatorname{tg}\left(\frac{\pi}{4} - \frac{ax}{2}\right) - \frac{1}{6a}\operatorname{tg}^3\left(\frac{\pi}{4} - \frac{ax}{2}\right) + C$

489. $\int \frac{dx}{(1 - \operatorname{sen} ax)^2} = \frac{1}{2a}\operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right) + \frac{1}{6a}\operatorname{tg}^3\left(\frac{\pi}{4} + \frac{ax}{2}\right) + C$

490. $\int \frac{dx}{1 + \operatorname{sen}^2 ax} = \frac{1}{2\sqrt{2a}}\operatorname{arc\,sen}\left(\frac{3\operatorname{sen}^2 ax - 1}{\operatorname{sen}^2 ax + 1}\right) + C$

491. $\int \frac{dx}{1 - \operatorname{sen}^2 ax} = \int \frac{dx}{\cos^2 ax} = \frac{1}{a}\operatorname{tg} ax + C$

492. $\int \frac{x\ dx}{1 + \operatorname{sen} ax} = -\frac{x}{a}\operatorname{tg}\left(\frac{\pi}{4} - \frac{ax}{2}\right) + \frac{2}{a^2}\ln\left|\operatorname{sen}\left(\frac{\pi}{4} + \frac{ax}{2}\right)\right| + C$

493. $\int \frac{x\ dx}{1 - \operatorname{sen} ax} = \frac{x}{a}\operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right) + \frac{2}{a^2}\ln\left|\operatorname{sen}\left(\frac{\pi}{4} - \frac{ax}{2}\right)\right| + C$

494. $\int \operatorname{sen} cx\ \operatorname{sen} dx\ dx = \frac{\operatorname{sen}(c-d)x}{2(c-d)} - \frac{\operatorname{sen}(c+d)x}{2(c+d)} + C$; (para $|c| \neq |d|$; si $|c| = |d|$, ver 468)

495. $\displaystyle\int \frac{dx}{c+d\ \operatorname{sen} ax} = \frac{2}{a\sqrt{c^2-d^2}} \operatorname{arc\,tg} \frac{c\ \operatorname{tg}\dfrac{ax}{2}+d}{\sqrt{c^2-d^2}} + C; \quad (\text{si } c^2 > d^2) =$

$$= \frac{1}{a\sqrt{d^2-c^2}} \ln \left| \frac{c\ \operatorname{tg}\dfrac{ax}{2}+d-\sqrt{d^2-c^2}}{b\ \operatorname{tg}\dfrac{ax}{2}+d\sqrt{d^2-c^2}} \right| + C; \quad (\text{si } c^2 < d^2)$$

496. $\displaystyle\int \frac{dx}{(c+d\ \operatorname{sen} ax)^2} = \frac{d\ \cos ax}{a\,(c^2-d^2)\ (c+d\ \operatorname{sen} ax)} + \frac{c}{c^2-d^2}\int \frac{dx}{c+d\ \operatorname{sen} ax}$; (ver 495)

497. $\displaystyle\int \frac{dx}{c^2+d^2\ \operatorname{sen}^2 ax} = \frac{1}{ac\sqrt{c^2+d^2}} \operatorname{arc\,tg} \frac{\sqrt{c^2+d^2}\operatorname{tg} ax}{c} + C; \ (c>0)$

498. $\displaystyle\int \frac{dx}{c^2-d^2\operatorname{sen}^2 ax} = \frac{1}{ac\sqrt{c^2-d^2}} \operatorname{arc\,tg} \frac{\sqrt{c^2-d^2}\operatorname{tg} ax}{c} + C; \quad (c^2>d^2; c>0) =$

$$= \frac{1}{2ac\sqrt{d^2-c^2}} \ln \left| \frac{\sqrt{d^2-c^2}\operatorname{tg} ax + c}{\sqrt{d^2-c^2}\operatorname{tg} ax - c} \right| + C; \quad (d^2>c^2; c>0)$$

499. $\displaystyle\int \frac{dx}{\operatorname{sen} ax\,(d+c\ \operatorname{sen} ax)} = \frac{1}{ac}\ln\left|\operatorname{tg}\frac{ax}{2}\right| - \frac{d}{c}\int \frac{dx}{c+d\ \operatorname{sen} ax}$; (ver 495)

500. $\displaystyle\int \frac{\operatorname{sen} ax\ dx}{c+d\ \operatorname{sen} ax} = \frac{x}{d} - \frac{c}{d}\int \frac{dx}{c+d\ \operatorname{sen} ax}$; (ver 495)

501. $\displaystyle\int \frac{\operatorname{sen} ax\ dx}{(c+d\ \operatorname{sen} ax)^2} = \frac{c\ \cos ax}{a\,(d^2-c^2)\ (d+c\ \operatorname{sen} ax)} + \frac{d}{d^2-c^2}\int \frac{dx}{c+d\ \operatorname{sen} ax}$; (ver 495)

a_2. Integrales en las que figura la función coseno

502. $\displaystyle\int \cos ax\ dx = \frac{\operatorname{sen} ax}{a} + C$

503. $\displaystyle\int \cos^2 ax\ dx = \frac{x}{2} + \frac{\operatorname{sen} 2ax}{4a} + C$

504. $\displaystyle\int \cos^3 ax\ dx = \frac{\operatorname{sen} ax}{a} - \frac{\operatorname{sen}^3 ax}{3a} + C$

505. $\int \cos^4 ax \; dx = \frac{3x}{8} + \frac{\operatorname{sen} 2ax}{4a} + \frac{\operatorname{sen} 4ax}{32a} + C$

506. $\int \cos^n ax \; dx = \frac{\cos^{n-1} ax \; \operatorname{sen} ax}{an} + \frac{n-1}{n} \int \cos^{n-2} ax \; dx$

507. $\int x \; \cos ax \; dx = \frac{\cos ax}{a^2} + \frac{x \; \operatorname{sen} ax}{a} + C$

508. $\int x \; \cos^2 ax \; dx = \frac{x^2}{4} + \frac{x \; \operatorname{sen} 2ax}{4a} + \frac{\cos 2ax}{8a^2} + C$

509. $\int x^2 \cos ax \; dx = \frac{2x}{a^2} \cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3} \right) \operatorname{sen} ax + C$

510. $\int x^3 \cos ax \; dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4} \right) \cos ax + \left(\frac{x^3}{a} - \frac{6x}{a^3} \right) \operatorname{sen} ax + C$

511. $\int x^n \cos ax \; dx = \frac{x^n \operatorname{sen} ax}{a} + \frac{n x^{n-1} \cos ax}{a^2} - \frac{n(n-1)}{a^2} \int x^{n-2} \cos ax \; dx$

512. $\int \frac{dx}{\cos ax} = \frac{1}{a} (\sec ax + \operatorname{tg} ax) + C$

513. $\int \frac{dx}{\cos^2 ax} = \frac{1}{a} \operatorname{tg} ax + C$

514. $\int \frac{dx}{\cos^3 ax} = \frac{\operatorname{sen} ax}{2a \; \cos^2 x} + \frac{1}{2a} \ln \operatorname{tg} \left(\frac{\pi}{4} + \frac{ax}{2} \right) + C$

515. $\int \frac{dx}{\cos^n ax} = \frac{\operatorname{sen} ax}{a(n-1) \cos^{n-1} ax} + \frac{n-2}{n-1} \int \frac{dx}{\cos^{n-2} ax}$

516. $\int \frac{\cos ax}{x} dx = \ln x - \frac{(ax)^2}{2.2!} + \frac{(ax)^4}{4.4!} - \frac{(ax)^6}{6.6!} + \ldots\ldots\ldots\ldots$

517. $\int \frac{\cos ax}{x^2} = - \frac{\cos ax}{x} - a \int \frac{\operatorname{sen} ax}{x} dx$ (ver 481)

518. $\int \frac{\cos ax}{x^n} dx = - \frac{\cos ax}{(n-1) x^{n-1}} - \frac{a}{n-1} \int \frac{\operatorname{sen} ax}{x^{n-1}} dx$ (ver 483)

519. $\int \frac{x \; dx}{\cos ax} = \frac{1}{a^2} \left[\frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \ldots + \frac{En\,(ax)^{2n+2}}{(2n+2)\;(2n)!} + \ldots \right]$

(siendo En número de Euler)

520. $\int \frac{x \; dx}{\cos^n ax} = \frac{x \; \operatorname{sen} ax}{a(n-1) \cos^{n-1} ax} - \frac{1}{a^2 (n-1)\;(n-2) \cos^{n-2} ax} + \frac{n-2}{n-1} \int \frac{x \; dx}{\cos^{n-2} ax}$

521. $\int \frac{dx}{1+\cos ax} = \frac{1}{a} \operatorname{tg} \frac{ax}{2} + C$

522. $\int \frac{dx}{1-\cos ax} = \frac{x}{a} \operatorname{cotg} \frac{ax}{2} + C$

523. $\int \frac{dx}{(1+\cos ax)^2} = \frac{1}{2a} \operatorname{tg} \frac{ax}{2} + \frac{1}{6a} \operatorname{tg}^3 \frac{ax}{2} + C$

524. $\int \frac{dx}{(1-\cos ax)^2} = -\frac{1}{2a} \operatorname{cotg} \frac{ax}{2} - \frac{1}{6a} \operatorname{cotg}^3 \frac{ax}{2} + C$

525. $\int \frac{dx}{1+\cos^2 x} = \frac{1}{2\sqrt{2}a} \operatorname{arc\,sen} \left(\frac{1-3\cos^2 ax}{1+\cos^2 ax} \right) + C$

526. $\int \frac{dx}{1-\cos^2 x} = \int \frac{dx}{\operatorname{sen}^2 ax} = -\frac{1}{a} \operatorname{cotg} ax + C$

527. $\int \frac{x\ dx}{1+\cos ax} = \frac{x}{a} \operatorname{tg} \frac{ax}{2} + \frac{2}{a^2} \ln \left| \cos \frac{ax}{2} \right| + C$

528. $\int \frac{x\ dx}{1-\cos ax} = -\frac{x}{a} \operatorname{cotg} \frac{ax}{2} + \frac{2}{a^2} \ln \left| \operatorname{sen} \frac{ax}{2} \right| + C$

529. $\int \cos cx\ \cos dx\ dx = \frac{\operatorname{sen}(c-d)x}{2(c-d)} + \frac{\operatorname{sen}(c+d)x}{2(c+d)}$; (para $|c| \neq |d|$; si $|c| = |d|$) (ver 503

530. $\int \frac{dx}{c+d\ \cos ax} = \frac{2}{a\sqrt{c^2-d^2}} \operatorname{arc\,tg} \frac{(c-d) \operatorname{tg} \frac{ax}{2}}{\sqrt{c^2-d^2}} + C$ (si $c^2 > d^2$) $=$

$$= \frac{1}{a\sqrt{d^2-c^2}} \ln \left| \frac{(d-c) \operatorname{tg} \frac{ax}{2} + \sqrt{d^2-c^2}}{(d-c) \operatorname{tg} \frac{ax}{2} - \sqrt{d^2-c^2}} \right| + C; \quad (\text{si } c^2 < d^2)$$

531. $\int \frac{dx}{(c+d\ \cos ax)^2} = \frac{d\ \operatorname{sen} ax}{a(d^2-c^2)(c+d\ \cos ax)} - \frac{b}{d^2-c^2} \int \frac{dx}{c+d\ \cos ax}$ (ver 530

532. $\int \frac{dx}{c^2+d^2\cos^2 ax} = \frac{1}{ac\sqrt{c^2+d^2}} \operatorname{arc\,tg} \frac{c\ \operatorname{tg} ax}{\sqrt{c^2+d^2}} + C$; (si $c > 0$)

533. $\int \frac{dx}{c^2-d^2\cos^2 ax} = \frac{1}{ac\sqrt{c^2-d^2}} \operatorname{arc\,tg} \frac{c\ \operatorname{tg} ax}{\sqrt{c^2-d^2}} + C$; (si $c^2 > d^2; c > 0$) $=$

$$= \frac{1}{2ac\sqrt{d^2 - c^2}} \ln \left| \frac{c \ \operatorname{tg} ax - \sqrt{d^2 - c^2}}{c \ \operatorname{tg} ax + \sqrt{d^2 - c^2}} \right| + C; \quad (\text{si } d^2 > c^2; c > 0)$$

534. $\int \frac{dx}{\cos ax\,(c + d \ \cos ax)} = \frac{1}{ac} \ln \left| \operatorname{tg}\left(\frac{ax}{2} + \frac{\pi}{4}\right) \right| - \int \frac{dx}{c + d \ \cos ax};$ (ver 530)

535. $\int \frac{\cos ax \ dx}{c + d \ \cos ax} = \frac{x}{d} - \frac{c}{d} \int \frac{dx}{c + d \ \cos ax};$ (ver 530)

a$_3$. Integrales en las que figuran las funciones seno y coseno

536. $\int \operatorname{sen} ax \ \cos ax \ dx = \frac{1}{2a} \operatorname{sen}^2 ax + C$

537. $\int \operatorname{sen}^2 ax \ \cos^2 ax \ dx = \frac{x}{8} - \frac{\operatorname{sen} 4ax}{32a} + C$

538. $\int \operatorname{sen}^n ax \ \cos ax \ dx = \frac{1}{a(n+1)} \operatorname{sen}^{n+1} ax + C;$ $(n \neq -1)$

539. $\int \operatorname{sen} ax \ \cos^n ax \ dx = -\frac{1}{a(n+1)} \cos^{n+1} ax + C;$ $(n \neq -1)$

540. $\int \frac{dx}{\operatorname{sen} ax \ \cos x} = \frac{1}{a} \ln |\operatorname{tg} ax| + C$

541. $\int \frac{dx}{\operatorname{sen}^2 ax \ \cos ax} = \frac{1}{a} \ln \operatorname{tg} \left| \frac{\pi}{4} + \frac{ax}{2} \right| - \frac{1}{\operatorname{sen} ax} + C$

542. $\int \frac{dx}{\operatorname{sen} ax \ \cos^2 ax} = \frac{1}{a} \ln \left| \operatorname{tg} \frac{ax}{2} \right| + \frac{1}{\cos ax} + C$

543. $\int \frac{dx}{\operatorname{sen}^3 ax \ \cos ax} = \frac{1}{a} \ln |\operatorname{tg} ax| - \frac{1}{2\operatorname{sen}^2 ax} + C$

544. $\int \frac{dx}{\operatorname{sen} ax \ \cos^3 ax} = \frac{1}{a} \ln |\operatorname{tg} ax| + \frac{1}{2\cos^2 x} + C$

545. $\int \frac{dx}{\operatorname{sen}^2 ax \ \cos^2 ax} = -\frac{2}{a} \operatorname{cotg} 2ax + C$

546. $\int \frac{dx}{\operatorname{sen}^2 ax \ \cos^3 ax} = \frac{1}{a} \frac{\operatorname{sen} ax}{2\cos^2 ax} - \frac{1}{\operatorname{sen} ax} + \frac{3}{2} \ln \left| \operatorname{tg} \frac{\pi}{4} + \frac{ax}{2} \right| + C$

547. $\int \frac{dx}{\operatorname{sen}^3 ax \cos^2 ax} = \frac{1}{a}\frac{1}{\cos ax} - \frac{\cos ax}{2\operatorname{sen}^2 ax} + \frac{3}{2}\ln\left|\operatorname{tg}\frac{ax}{2}\right| + C$

548. $\int \frac{dx}{\operatorname{sen} ax \cos^n ax} = \frac{1}{a(n-1)\cos^{n-1} ax} + \int \frac{dx}{\operatorname{sen} ax \cos^{n-2} ax}; \qquad (n \neq -1)$

549. $\int \frac{dx}{\operatorname{sen}^n ax \cos ax} = -\frac{1}{a(n-1)\operatorname{sen}^{n-1} ax} + \int \frac{dx}{\operatorname{sen}^{n-2} ax \cos ax}$

550. $\int \frac{dx}{\operatorname{sen}^n ax \cos^m ax} = -\frac{1}{a(n-1)}\frac{1}{\operatorname{sen}^{n-1} ax \cos^{m-1} ax} +$

$+ \frac{n+m-2}{n-1}\int \frac{dx}{\operatorname{sen}^{n-2} ax \cos^m ax}; \qquad (m > 0; n > 1) =$

$= \frac{1}{a(m-1)}\frac{1}{\operatorname{sen}^{n-1} ax \cos^{m-1} ax} +$

$+ \frac{n+m-2}{m-1}\int \frac{dx}{\operatorname{sen}^{n} ax \cos^{m-2} ax}; \qquad (n > 0; m > 1)$

551. $\int \frac{\operatorname{sen} ax \, dx}{\cos^2 ax} = \frac{1}{a \cos ax} + C$

552. $\int \frac{\operatorname{sen}^2 ax \, dx}{\cos ax} = -\frac{1}{a}\operatorname{sen} ax + \frac{1}{a}\ln\left|\operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right)\right| + C$

553. $\int \frac{\operatorname{sen}^2 ax \, dx}{\cos^n ax} = \frac{\operatorname{sen} ax}{a(n-1)\cos^{n-1} ax} - \frac{1}{n-1}\int \frac{dx}{\cos^{n-2} ax}; \qquad (n \neq 1)$

554. $\int \frac{\operatorname{sen} ax \, dx}{\cos^3 ax} = \frac{1}{2a \cos^2 ax} + C$

555. $\int \frac{\operatorname{sen} ax \, dx}{\cos^n ax} = \frac{1}{a(n-1)\cos^{n-1} ax} + C$

556. $\int \frac{\operatorname{sen}^2 ax \, dx}{\cos^n ax} = \frac{\operatorname{sen} ax}{a(n-1)\cos^{n-1} ax} - \frac{1}{n-1}\int \frac{dx}{\cos^{n-2} ax}; \qquad (n \neq 1)$

557. $\int \frac{\operatorname{sen}^3 ax \, dx}{\cos ax} = -\frac{1}{a}\frac{\operatorname{sen}^2 ax}{2} + \ln|\cos ax| + C$

558. $\int \frac{\operatorname{sen}^3 ax \, dx}{\cos^2 ax} = \frac{1}{a}\cos ax + \frac{1}{\cos ax} + C$

559. $\int \frac{\operatorname{sen}^3 ax \, dx}{\cos^n ax} = \frac{1}{a}\left[\frac{1}{(n-1)\cos^{n-1} ax} - \frac{1}{(n-3)\cos^{n-3} ax}\right] + C; \qquad (n \neq 1; n \neq 3)$

560. $\displaystyle\int \frac{\operatorname{sen}^n ax}{\cos ax}\,dx = -\frac{\operatorname{sen}^{n-1} ax}{a(n-1)} + \int \frac{\operatorname{sen}^{n-2} ax\;\,dx}{\cos ax};$ $\qquad (n \neq 1)$

561. $\displaystyle\int \frac{\cos ax\;\,dx}{\operatorname{sen}^2 ax} = -\frac{1}{a\;\operatorname{sen} ax} + C$

562. $\displaystyle\int \frac{\cos ax\;\,dx}{\operatorname{sen}^3 ax} = -\frac{1}{2a\;\operatorname{sen}^2 ax} + C$

563. $\displaystyle\int \frac{\cos ax\;\,dx}{\operatorname{sen}^n ax} = -\frac{1}{a(n-1)\operatorname{sen}^{n-1} ax} + C$

564. $\displaystyle\int \frac{\cos^2 ax\;\,dx}{\operatorname{sen} ax} = \frac{1}{a}\cos ax + \ln\left|\operatorname{tg}\frac{ax}{2}\right| + C$

565. $\displaystyle\int \frac{\cos^2 ax\;\,dx}{\operatorname{sen}^3 ax} = -\frac{1}{2a}\frac{\cos ax}{\operatorname{sen}^2 ax} - \ln\left|\operatorname{tg}\frac{ax}{2}\right| + C$

566. $\displaystyle\int \frac{\cos^2 ax\;\,dx}{\operatorname{sen}^n ax} = -\frac{1}{(n-1)}\frac{\cos ax}{a\;\operatorname{sen}^{n-1} ax} + \int \frac{dx}{\operatorname{sen}^{n-2} ax};$ $\qquad (n \neq 1)$

567. $\displaystyle\int \frac{\cos^3 ax\;\,dx}{\operatorname{sen} ax} = \frac{1}{a}\frac{\cos^2 ax}{2} + \ln|\operatorname{sen} ax| + C$

568. $\displaystyle\int \frac{\cos^3 ax\;\,dx}{\operatorname{sen}^2 ax} = -\frac{1}{a}\operatorname{sen} ax + \frac{1}{\operatorname{sen} ax} + C$

569. $\displaystyle\int \frac{\cos^n ax}{\operatorname{sen} ax}\,dx = \frac{\cos^{n-1} ax}{a(n-1)} + \int \frac{\cos^{n-2} ax\;\,dx}{\operatorname{sen} ax};$ $\qquad (n \neq 1)$

a$_4$. Integrales en las que figura la función tangente

570. $\displaystyle\int \operatorname{tg} ax\;\,dx = -\frac{1}{a}\ln\cos|ax| + C$

571. $\displaystyle\int \operatorname{tg}^2 ax\;\,dx = \frac{\operatorname{tg} ax}{3} - x + C$

572. $\displaystyle\int \operatorname{tg}^3 ax\;\,dx = \frac{\operatorname{tg}^2 ax}{2a} + \frac{1}{a}\ln|\cos ax| + C$

573. $\displaystyle\int x\;\operatorname{tg} ax\;\,dx = \frac{ax^3}{3} + \frac{a^3x^5}{15} + \frac{2a^5x^7}{105} + \frac{17a^7x^9}{2835} + \ldots + \frac{2^{2n}(2^{2n}-1)Bn\;\,a^{2n-1}x^{2n+1}}{(2n+1)!} + \ldots;$

Bn: número de Bernoulli

574. $\int \frac{dx}{\operatorname{tg} ax} = \frac{1}{a} \ln|\operatorname{sen} ax| + C$

575. $\int x \operatorname{tg}^2 ax \, dx = \frac{x \operatorname{tg} ax}{a} + \frac{1}{a^2} \ln|\cos ax| - \frac{x^2}{2} + C$

576. $\int \operatorname{tg}^n ax \, dx = \frac{\operatorname{tg}^{n-1} ax}{(n-1)a} - \int \operatorname{tg}^{n-2} ax \, dx$

577. $\int \frac{\operatorname{tg} ax \, dx}{x} = ax + \frac{(ax)^3}{9} + \frac{2(ax)^5}{75} + \frac{17(ax)^7}{2205} + \ldots.. + \frac{2^{2n}(2^{2n}-1) Bn \, (ax) \, 2^{n-1}}{(2n-1)\,(2n)!} + .$

a$_5$. Integrales en las que figura la función cotangente

578. $\int \operatorname{cotg} ax \, dx = \frac{1}{a} \ln|\operatorname{sen} ax| + C$

579. $\int \operatorname{cotg}^2 ax \, dx = -\frac{\operatorname{cotg} ax}{a} - x + C$

580. $\int \operatorname{cotg}^3 ax \, dx = -\frac{\operatorname{cotg}^2 ax}{2a} - \frac{1}{a} \ln|\operatorname{sen} ax| + C$

581. $\int x \operatorname{cotg} ax \, dx = \frac{x}{a} - \frac{ax^3}{9} - \frac{a^3 x^5}{225} - \ldots.. - \frac{2^{2n} Bn \; a^{2n-1} x^{2n+1}}{(2n+1)!}$

582. $\int \frac{dx}{\operatorname{cotg} ax} = -\frac{1}{a} \ln|\cos ax| + C$

583. $\int x \operatorname{cotg}^2 ax \, dx = \frac{x \operatorname{cotg} ax}{a} + \frac{1}{a^2} \ln|\operatorname{sen} ax| - \frac{x^2}{2}$

584. $\int \operatorname{cotg}^n ax \, dx = -\frac{\operatorname{cotg}^{n-1} ax}{(n-1)a} - \int \operatorname{cotg}^{n-2} ax \, dx$

585. $\int \frac{\operatorname{cotg} ax \, dx}{x} = -\frac{1}{ax} - \frac{(ax)}{3} - \frac{(ax)^3}{135} - \ldots.. - \frac{2^{2n} Bn \, (ax)^{2n-1}}{(2n-1)\,(2n)!} - \ldots..$

a$_6$. Integrales en las que figura la función secante

586. $\int \sec ax \, dx = \frac{1}{a} \ln|\sec ax + \operatorname{tg} ax| + C$

587. $\int \sec^2 ax \; dx = \frac{\operatorname{tg} ax}{a} + C$

588. $\int \sec^3 ax \; dx = -\frac{\sec ax \; \operatorname{tg} ax}{2a} + \frac{1}{2a} \ln|\sec ax + \operatorname{tg} ax| + C$

589. $\int \sec^n ax \; dx = \frac{\sec^{n-2} ax \; \operatorname{tg} ax}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax \; dx$

590. $\int \frac{dx}{\sec ax} = \frac{\operatorname{sen} ax}{a} + C$

a$_7$. Integrales en las que figura la función cosecante

591. $\int \operatorname{cosec} ax \; dx = \frac{1}{a} \ln \left| \operatorname{tg} \frac{ax}{2} \right| + C$

592. $\int \operatorname{cosec}^2 ax \; dx = -\frac{\operatorname{cotg} ax}{a} + C$

593. $\int \operatorname{cosec}^3 ax \; dx = -\frac{\operatorname{cosec} ax \; \operatorname{cotg} ax}{2a} + \frac{1}{2a} \ln \left| \operatorname{tg} \frac{ax}{2} \right| + C$

594. $\int \operatorname{cosec}^n ax \; dx = -\frac{\operatorname{cosec}^{n-2} ax \; \operatorname{cotg} ax}{a(n-1)} + \frac{n-2}{n-1} \int \operatorname{cosec}^{n-2} ax \; dx$

595. $\int \frac{dx}{\operatorname{cosec} x} = -\frac{\cos ax}{a} + C$

b. Integrales de funciones trigonométricas inversas

596. $\int \operatorname{arc\,sen} \frac{x}{a} dx = x \; \operatorname{arc\,sen} \frac{x}{a} + \sqrt{a^2 - x^2} + C$

597. $\int x \; \operatorname{arc\,sen} \frac{x}{a} dx = \left(\frac{x^2}{2} - \frac{a^2}{2}\right) \operatorname{arc} sen \frac{x}{a} + \frac{x}{4} \sqrt{a^2 - x^2} + C$

598. $\int x^2 \operatorname{arc\,sen} \frac{x}{a} dx = \frac{x^3}{3} \operatorname{arc\,sen} \frac{x}{a} + \frac{1}{9} (x^2 + 2a^2) \sqrt{a^2 - x^2} + C$

599. $\int x^n \text{arc sen}\frac{x}{a}dx = \frac{x^{n+1}}{n+1}\text{arc sen}\frac{x}{a} - \frac{1}{n+1}\int\frac{x^{n+1}}{\sqrt{a^2-x^2}}dx$

600. $\int(\text{arc sen}\frac{x}{a})^2 dx = x(\text{arc sen}\frac{x}{a})^2 - 2x + 2\sqrt{a^2-x^2}\,\text{arc sen}\frac{x}{a} + C$

601. $\int \text{arc cos}\frac{x}{a}dx = x\ \text{arc cos}\frac{x}{a} - \sqrt{a^2-x^2} + C$

602. $\int x\ \text{arc cos}\frac{x}{a}dx = (\frac{x^2}{2} - \frac{a^2}{4})\,\text{arc cos}\frac{x}{a} - \frac{x\sqrt{a^2-x^2}}{4} + C$

603. $\int x^2\text{arc cos}\frac{x}{a}dx = \frac{x^3}{3}\text{arc cos}\frac{x}{a} - \frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2} + C$

604. $\int x^n\text{arc cos}\frac{x}{a}dx = \frac{x^{n+1}}{n+1}\text{arc cos}\frac{x}{a} + \frac{1}{n+1}\int\frac{x^{n+1}}{\sqrt{a^2-x^2}}$

605. $\int(\text{arc cos}\frac{x}{a})^2 dx = x(\text{arc cos}\frac{x}{a})^2 - 2x - 2\sqrt{a^2-x^2}\,\text{arc cos}\frac{x}{a} + C$

606. $\int \text{arc tg}\frac{x}{a}dx = x\ \text{arc tg}\frac{x}{a} - \frac{a}{2}\ln|x^2+a^2| + C$

607. $\int x\ \text{arc tg}\frac{x}{a}dx = \frac{1}{2}(x^2+a^2)\,\text{arc tg}\frac{x}{a} - \frac{ax}{2} + C$

608. $\int x^2\text{arc tg}\frac{x}{a}dx = \frac{x^3}{3}\text{arc tg}\frac{x}{a} - \frac{ax^2}{6} + \frac{a^2}{6}\ln|x^2+a^2| + C$

609. $\int x^n\text{arc tg}\frac{x}{a}dx = \frac{x^{n+1}}{n+1}\text{arc tg}\frac{x}{a} - \frac{a}{n+1}\int\frac{x^{n+1}}{x^2+a^2}dx$

610. $\int \text{arc cotg}\frac{x}{a}dx = x\ \text{arc cotg}\frac{x}{a} + \frac{a}{2}\ln|x^2+a^2| + C$

611. $\int x\ \text{arc cotg}\frac{x}{a}dx = \frac{1}{2}(x^2+a^2)\,\text{arc cotg}\frac{x}{a} + \frac{ax}{2} + C$

612. $\int x^2\text{arc cotg}\frac{x}{a}dx = \frac{x^3}{3}\text{arc cotg}\frac{x}{a} + \frac{ax^2}{6} - \frac{a^3}{6}\ln|x^2+a^2| + C$

613. $\int x^n\text{arc cotg}\frac{x}{a}dx = \frac{x^{n+1}}{n+1}\text{arc cotg}\frac{x}{a} + \frac{a}{n+1}\int\frac{x^{n+1}}{x^2+a^2}dx;\qquad (n \neq -1)$

614. $\int \text{arc sec}\frac{x}{a}dx = x\ \text{arc sec}\frac{x}{a} - a\ \ln\left|x+\sqrt{x^2-a^2}\right| + C;\qquad (0 < \text{arc sec}\frac{x}{a} < \frac{\pi}{2}) =$

$$= x \ \text{arc sec}\frac{x}{a} + a \ \ln\left|x + \sqrt{x^2 - a^2}\right| + C; \qquad (\frac{\pi}{2} < \text{arc sec}\frac{x}{a} < \pi)$$

615. $\int x \ \text{arc sec}\frac{x}{a}dx = \frac{x^2}{2}\text{arc sec}\frac{x}{a} - \frac{a}{2}\sqrt{x^2 - a^2} + C; \qquad (0 < \text{arc sec}\frac{x}{a} < \frac{\pi}{2}) =$

$$= \frac{x}{2}\text{arc sec}\frac{x}{a} + \frac{a}{2}\sqrt{x^2 - a^2} + C; \qquad (\frac{\pi}{2} - \text{arc sec}\frac{a}{x} < \pi)$$

616. $\int \text{arc cosec}\frac{x}{a}dx = x \ \text{arc cosec}\frac{x}{a} + a \ \ln\left|x + \sqrt{x^2 - a^2}\right| + C; \quad (0 < \text{arc cosec}\frac{x}{a} < \frac{\pi}{2}) =$

$$= x \ \text{arc cosec}\frac{x}{a} - a \ \ln\left|x + \sqrt{x^2 - a^2}\right| + C; \quad (-\frac{\pi}{2} < \text{arc cosec}\frac{x}{a} < 0)$$

617. $\int x \ \text{arc cosec}\frac{x}{a}dx = \frac{x^2}{2}\text{arc cosec}\frac{x}{a} + \frac{a}{2}\sqrt{x^2 - a^2} + C; \quad (0 < \text{arc cosec}\frac{x}{a} < \frac{\pi}{2}) =$

$$= \frac{x^2}{2}\text{arc cosec}\frac{x}{a} - \frac{a}{2}\sqrt{x^2 - a^2} + C; \quad (-\frac{\pi}{2} < \text{arc cosec}\frac{x}{a} < 0)$$

c. Integrales de funciones logarítmicas

618. $\int \frac{\ln x}{x^2}dx = -\frac{\ln x}{x} - \frac{1}{x} + C$

619. $\int (\ln x)^2 dx = x(\ln|x|)^2 - 2x \ \ln|x| + 2x + C$

620. $\int (\ln x)^2 dx = x(\ln|x|)^2 - 3x(\ln|x|)^2 + 6x \ \ln|x| - 6x + C$

621. $\int (\ln x)^n dx = x[\ln|x|]^n - n\int [\ln|x|]^{n-1}dx$

622. $\int \frac{dx}{\ln x} = \ln|(\ln|x|)| + \ln|x| + \frac{(\ln|x|)^2}{2.2!} + \frac{(\ln|x|)^3}{3.3!} + \ldots..$

623. $\int \frac{dx}{x \ \ln x} = \ln|(\ln|x|)| + C$

624. $\int \frac{dx}{x(\ln x)^n} = -\frac{1}{(n-1)\ (\ln|x|)^{n-1}} + C; \qquad (n \neq 1)$

625. $\int \frac{x^n dx}{\ln x} = \ln|(\ln|x|)| + \frac{n+1}{1.1!}\ln|x| + \frac{(n+1)^2(\ln|x|)^2}{2.2!} + \ldots.. + C$

626. $\int \frac{x^m dx}{(\ln x)^n} = -\frac{x^{m+1}}{(n-1)\ (\ln|x|)^{n-1}} + \frac{m+1}{n-1}\int \frac{x^m dx}{(\ln|x|)^{n-1}}; \qquad (n \neq 1)$

d. Integrales de funciones exponenciales

627. $\int e^{ax}dx = \frac{1}{a}e^{ax} + C$

628. $\int x\ e^{ax}dx = \frac{e^{ax}}{a}(x - \frac{1}{a}) + C$

629. $\int x^2 e^{ax}dx = \frac{e^{ax}}{a}(x^2 - \frac{2x}{a} + \frac{2}{a^2}) + C$

630. $\int x^3 e^{ax^2}dx = \frac{1}{2a}x^2 e^{ax^2} - \frac{1}{2a^2}e^{ax^2} + C$

631. $\int \frac{e^{ax}}{x}dx = \ln|x| + \frac{ax}{1.1!} + \frac{(ax)^2}{2.2!} + \frac{(ax)^3}{3.3!} + ... + C$

632. $\int e^{ax}\ln x\ dx = \frac{e^{ax}\ln x}{a} - \frac{1}{a}\int \frac{e^{ax}}{x}dx$

633. $\int e^{ax}\operatorname{sen} cx\ dx = \frac{e^{ax}}{a^2 + c^2}(a\ \operatorname{sen} cx - c\ \cos cx) + C$

634. $\int e^{ax}\cos cx\ dx = \frac{e^{ax}}{a^2 + c^2}(a\ \cos cx + c\ \operatorname{sen} cx) + C$

635. $\int \frac{dx}{1 + e^{ax}} = \frac{1}{a}\ln\left|\frac{e^{ax}}{1 + e^{ax}}\right| + C$

636. $\int \frac{dx}{c + d\ e^{ax}} = \frac{x}{c} - \frac{1}{ac}(c + d\ e^{ax}) + C$

637. $\int \frac{e^{ax}dx}{c + d\ e^{ax}} = \frac{1}{ad}\ln|c + d\ e^{ax}| + C$

e. Integrales de funciones hiperbólicas

638. $\int sh\ ax\ dx = \frac{ch\ ax}{a} + C$

639. $\int ch\ ax\ dx = \frac{sh\ ax}{a} + C$

640. $\int sh\ ax\ ch\ ax\ dx = \frac{sh^2 ax}{2a} + C$

641. $\int \text{tg}\,h\ ax\ dx = \frac{1}{a}\ln|ch\ ax| + C$

642. $\int \text{cotg}\,h\ ax\ dx = \frac{1}{a}\ln|sh\ ax| + C$

643. $\int \sec h\ ax\ dx = \frac{2}{a}\text{arc tg}\,e^{ax} + C$

644. $\int \text{cosec}\,h\ ax\ dx = \frac{1}{a}\ln\left|\text{tg}\,h\frac{ax}{2}\right| + C$

645. $\int sh^2 ax\ dx = -\frac{x}{2} + \frac{sh\ ax\ ch\ ax}{2a} + C$

646. $\int ch^2 ax\ dx = \frac{x}{2} + \frac{sh\ ax\ ch\ ax}{2} + C$

647. $\int sh^2 ax\ ch^2\ ax\ dx = -\frac{x}{8} + \frac{sh\ 4ax}{32a} + C$

648. $\int \text{tg}\,h^2 ax\ dx = -\frac{\text{tg}\,h\ ax}{a} + x + C$

649. $\int \text{cotg}\,h^2 ax\ dx = -\frac{\text{cotg}\,h\ ax}{a} + x + C$

650. $\int \sec h^2 ax\ dx = \frac{\text{tg}\,h\ ax}{a} + C$

651. $\int \text{cosec}\,h^2 ax\ dx = -\frac{\text{cotg}\,h\ ax}{a} + C$

652. $\int \frac{dx}{sh\ ax} = \frac{1}{a}\ln\left|\text{tg}\,h\frac{ax}{2}\right| + C$

653. $\int \frac{dx}{ch\ ax} = \frac{2}{a}\text{arc tg}\,e^{ax} + C$

654. $\int \frac{dx}{sh\ ax\ ch\ ax} = \frac{1}{a}\ln|\text{tg}\,h\ ax| + C$

655. $\int \frac{dx}{\text{tg}\,h\ ax} = \frac{1}{a}\ln|sh\ ax| + C$

656. $\int \frac{dx}{\text{cotg}\,h\ ax} = \frac{1}{a}\ln|ch\ ax| + C$

657. $\int \frac{dx}{\sec ax} = \frac{sh\ ax}{a} + C$

658. $\int \frac{dx}{\operatorname{cosec} h\ ax} = \frac{ch\ ax}{a} + C$

659. $\int x\ sh\ ax\ dx = -\frac{sh\ ax}{a^2} + \frac{x\ ch\ ax}{a} + C$

660. $\int x\ ch\ ax\ dx = -\frac{ch\ ax}{a^2} + \frac{x\ sh\ ax}{a} + C$

661. $\int x\ \operatorname{tg} ax\ dx = \frac{1}{a^2}\left(\frac{(ax)^3}{3} - \frac{(ax)^5}{15} + \frac{2(ax)^7}{105} - \ldots\right.$

$\left.\ldots + \frac{(-1)^{n-1}2^{2n}(2^{2n}-1)Bn(ax)^{2n+1}}{(2n+1)!} + \ldots\right] + C$ Bn: número de Bernou

662. $\int x\ \operatorname{cotg} h\ ax\ dx = \frac{1}{a^2}\left[ax + \frac{(ax)^3}{9} - \frac{(ax)^5}{225} + \ldots + \frac{(-1)^{n-1}2^{2n}Bn(ax)^{2n+1}}{(2n+1)!} + \ldots\right] + C$

Bn: número de Bernou

663. $\int x\ \sec h\ dx = \frac{1}{a^2}\left[\frac{(ax)^2}{2} - \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} - \ldots + \frac{(-1)^n En(ax)^{2n+2}}{(2n+2)(2n)!} + \ldots\right] + C$

En: número de Euler

664. $\int x\ \operatorname{cosec} h\ ax\ dx = \frac{1}{a^2}\left(ax - \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} - \ldots\right.$

$\left.\ldots + \frac{(-1)^n 2(2^{2n-1}-1)Bn(ax)^{2n+1}}{(2n+1)!} + \ldots\right] + C$ Bn: número de Bernoulli

665. $\int x^2 sh\ ax\ dx = -\frac{2x}{a^2} sh\ ax + \left(\frac{2}{a^2} + \frac{x^2}{a}\right) ch\ ax + C$

666. $\int x^2 ch\ ax\ dx = -\frac{2x}{a^2} ch\ ax + \left(\frac{x^2}{a} + \frac{2}{a^3}\right) sh\ ax + C$

667. $\int \frac{sh\ ax}{x} dx = ax + \frac{(ax)^3}{3.3!} + \frac{(ax)^5}{5.5!} + \ldots\ldots + C$

668. $\int \frac{ch\ ax}{x} dx = \ln|x| + \frac{(ax)^2}{2.2!} + \frac{(ax)^4}{4.4!} + \frac{(ax)^6}{6.6!} + \ldots\ldots + C$

669. $\int \frac{\operatorname{tg} h\ ax}{x} dx = ax - \frac{(ax)^3}{9} + \frac{2(ax)^6}{75} - \ldots\ldots + C$

670. $\int \frac{\operatorname{cotg} h\ ax}{x} dx = -\frac{1}{ax} + \frac{ax}{3} - \frac{(ax)^3}{135} + \ldots\ldots + C$

671. $\int \frac{\sec h \ ax}{x} dx = \ln|x| - \frac{(ax)^2}{4} + \frac{5(ax)^4}{96} - \frac{61(ax)^6}{4320} + \ldots\ldots + C$

672. $\int \frac{\operatorname{cosec} h \ ax}{x} dx = -\frac{1}{ax} - \frac{ax}{6} + \frac{7(ax)^3}{1080} - \ldots\ldots + C$

673. $\int \frac{sh \ ax}{x^2} dx = -\frac{sh \ ax}{x} + a\int \frac{ch \ ax}{x} dx$ (ver 667)

674. $\int \frac{ch \ ax}{x^2} dx = -\frac{ch \ ax}{x} + a\int \frac{sh \ ax}{x} dx$ (ver 666)

675. $\int \frac{dx}{sh^2 ax \ ch \ ax} = -\frac{1}{a} \operatorname{arc\,tg} sh \ ax - \frac{ch \ ax}{a} + C$

676. $\int \operatorname{tg} h^3 ax \ dx = -\frac{\operatorname{tg} h^2 ax}{2a} + \frac{1}{a} \ln|ch \ ax| + C$

677. $\int \operatorname{cotg} h^3 ax \ dx = -\frac{\operatorname{cotg} h^2 ax}{2a} + \frac{1}{a} \ln|sh \ ax| + C$

678. $\int \sec h^3 ax \ dx = \frac{\sec h \ ax \ \operatorname{tg} h \ ax}{2a} + \frac{1}{2a} \operatorname{arc\,tg} sh \ ax + C$

679. $\int \operatorname{cosec} h^3 ax \ dx = -\frac{\operatorname{cosec} h \ ax \ \operatorname{cotg} h \ ax}{2a} - \frac{1}{2a} \ln\left|\operatorname{tg} h \frac{ax}{2}\right| + C$

680. $\int x \ sh^2 ax \ dx = -\frac{x^2}{4} + \frac{x \ sh \ 2ax}{4a} - \frac{ch \ 2ax}{8a^2} + C$

681. $\int x \ ch^2 ax \ dx = \frac{x^2}{4} + \frac{x \ sh \ 2ax}{4a} - \frac{ch \ 2ax}{8a^2} + C$

682. $\int \frac{dx}{sh \ ax \ ch^2 ax} = \frac{1}{a} \ln\left|\operatorname{tg} h \frac{ax}{2}\right| + \frac{\sec h \ ax}{a} + C$

683. $\int \frac{dx}{sh^2 ax \ ch \ ax} = -\frac{2 \operatorname{cotg} h \ 2ax}{a} + C$

684. $\int \frac{sh^2 ax}{ch \ ax} dx = -\frac{1}{a} \operatorname{arc\,tg} sh \ ax + \frac{sh \ ax}{a} + C$

685. $\int \frac{ch^2 ax}{sh \ ax} dx = \frac{1}{a} \ln\left|\operatorname{tg} h \frac{ax}{2}\right| + \frac{ch \ ax}{a} + C$

686. $\int x \ \operatorname{tg} h^2 ax \ dx = -\frac{x \ \operatorname{tg} h \ ax}{a} + \frac{1}{a^2} \ln\left|ch \ ax + \frac{x^2}{2}\right| + C$

687. $\int x \ \text{cotg}\, h^2 ax \ dx = -\frac{x \ \text{cotg}\, h \ ax}{a} + \frac{1}{a^2}\ln\left|sh \ ax + \frac{x^2}{2}\right| + C$

688. $\int x \ \sec h^2 ax \ dx = \frac{x}{a}\text{tg}\, h \ ax - \frac{1}{a^2}\ln|ch \ ax| + C$

689. $\int x \ \text{cosec}\, h^2 ax \ dx = -\frac{x}{2}\text{cotg}\, h \ ax + \frac{1}{a^2}\ln|sh \ ax| + C$

690. $\int \frac{dx}{sh^2 ax} = -\frac{1}{a}\text{cotg}\, h \ ax + C$

691. $\int \frac{dx}{ch^2 ax} = \frac{1}{a}\text{tg}\, h \ ax + C$

f. Integrales de funciones hiperbólicas inversas

692. $\int \arg sh\frac{x}{a}dx = x \ \arg sh\frac{x}{a} - \sqrt{a^2+x^2} + C$

693. $\int \arg ch\frac{x}{a}dx = x \ \arg ch\frac{x}{a} - \sqrt{x^2-a^2} + C; \qquad (\arg ch\frac{x}{a} > 0) =$

$= x \ \arg ch\frac{x}{a} + \sqrt{x^2-a^2} + C; \qquad (\arg ch\frac{x}{a} < 0)$

694. $\int \arg \text{tg}\, h\frac{x}{a}dx = x \ \arg\text{tg}\, h\frac{x}{a} + \frac{a}{2}\ln|a^2-x^2| + C$

695. $\int \arg \text{cotg}\, h\frac{x}{a}dx = x \ \arg\text{cotg}\, h\frac{x}{a} + \frac{a}{2}\ln|x^2-a^2| + C$

696. $\int \arg \sec h\frac{x}{a}dx = x \ \arg\sec h\frac{x}{a} + a \ \arg sh\,(\frac{x}{a}) + C; \qquad (\sec h\frac{x}{a} > 0) =$

$= x \ \arg\sec h\frac{x}{a} - a \ \arg sh\,(\frac{x}{a}) + C; \qquad (\sec h\frac{x}{a} < 0)$

697. $\int \arg \text{cosec}\, h\frac{x}{a}dx = x \ \text{arc cosec}\, h\frac{x}{a} + a \ \arg sh\frac{x}{a} + C; \qquad (x > 0) =$

$= x \ \text{arc osec}\, h\frac{x}{a} - a \ \arg sh\frac{x}{a} + C; \qquad (x < 0)$

698. $\int x \ \arg sh\frac{x}{a}dx = (\frac{x^2}{2} + \frac{a^2}{4})\arg sh\frac{x}{a} - \frac{x\sqrt{a^2+x^2}}{4} + C$

699. $\int x \arg ch\frac{x}{a}dx = (\frac{x^2}{2} - \frac{a^2}{4})\arg ch\frac{x}{a} - \frac{x\sqrt{x^2-a^2}}{4} + C; \qquad (\arg ch\frac{x}{a} > 0) =$

$= (\frac{x^2}{2} - \frac{a^2}{4})\arg ch\frac{x}{a} + \frac{x\sqrt{x^2-a^2}}{4} + C; \qquad (\arg ch\frac{x}{a} < 0)$

700. $\int x \arg \operatorname{tg} h\frac{x}{a}dx = \frac{1}{2}(x^2 - a^2)\arg \operatorname{tg} h\frac{x}{a} + \frac{ax}{2} + C$

701. $\int x \arg \operatorname{cotg} h\frac{x}{a}dx = \frac{1}{2}(x^2 - a^2)\arg \operatorname{cotg}\frac{x}{a} + \frac{ax}{2} + C$

702. $\int x \arg \sec h\frac{x}{a}dx = \frac{x^2}{2}\arg \sec h\frac{x}{a} - \frac{a\sqrt{a^2-x^2}}{2} + C; \qquad (\arg \sec h\frac{x}{a} > 0) =$

$= \frac{x^2}{2}\arg \sec h\frac{x}{a} + \frac{a\sqrt{a^2-x^2}}{2} + C; \qquad (\arg \sec h\frac{x}{a} < 0)$

703. $\int x \arg \operatorname{cosec} h\frac{x}{a}dx = \frac{x^2}{2}\arg \operatorname{cosec} h\frac{x}{a} + \frac{a\sqrt{x^2+a^2}}{2} + C; \qquad (x > 0) =$

$= \frac{x^2}{2}\arg \operatorname{cosec} h\frac{x}{a} - \frac{a\sqrt{x^2+a^2}}{2} + C; \qquad (x < 0)$

704. $\int x^2 \arg sh\frac{x}{a}dx = \frac{x^3}{3}\arg sh\frac{x}{a} + \frac{(-x^2+2a^2)\sqrt{a^2+x^2}}{9} + C$

705. $\int x^2 \arg ch\frac{x}{a}dx = \frac{x^3}{3}\arg ch\frac{x}{a} - \frac{(x^2+2a^2)\sqrt{x^2-a^2}}{9} + C; \qquad (\arg ch\frac{x}{a} > 0) =$

$= \frac{x^3}{3}\arg ch\frac{x}{a} + \frac{(x^2+2a^2)\sqrt{x^2-a^2}}{9} + C; \qquad (\arg ch\frac{x}{a} < 0)$

706. $\int x^2 \arg \operatorname{tg} h\frac{x}{a}dx = \frac{x^3}{3}\arg \operatorname{tg} h\frac{x}{a} + \frac{ax^2}{6} + \frac{a^2}{6}\ln(a^2 - x^2) + C$

707. $\int x^2 \arg \operatorname{cotg} h\frac{x}{a}dx = \frac{ax^2}{6} + \frac{x^3}{3}\operatorname{arc\,cotg} h\frac{x}{a} + \frac{a^2}{6}\ln|x^2 - a^2| + C$

708. $\int x^n \arg sh\frac{x}{a}dx = \frac{x^{n+1}}{n+1}\arg sh\frac{x}{a} - \frac{1}{n+1}\int\frac{x^{n+1}}{\sqrt{a^2+x^2}}dx$

709. $\int x^n \arg ch\frac{x}{a}dx = \frac{x^{n+1}}{n+1}\arg ch\frac{x}{a} - \frac{1}{n+1}\int\frac{x^{n+1}}{\sqrt{x^2-a^2}}dx;$ $\quad (\arg ch\frac{x}{a} > 0) =$

$= \frac{x^{n+1}}{n+1}\arg ch\frac{x}{a} + \frac{1}{n+1}\int\frac{x^{n+1}}{\sqrt{x^2-a^2}}dx;$ $\quad (\arg ch\frac{x}{a} < 0)$

710. $\int x^n \arg \operatorname{tg} h\frac{x}{a}dx = \frac{x^{n+1}}{n+1}\arg \operatorname{tg} h\frac{x}{a} - \frac{a}{m+1}\int\frac{x^{n+1}}{a^2-x^2}dx$

711. $\int x^n \arg \operatorname{cotg} h\frac{x}{a}dx = \frac{x^{n+1}}{n+1}\arg \operatorname{cotg} h\frac{x}{a} - \frac{a}{m+1}\int\frac{x^{n+1}}{a^2-x^2}dx$

712. $\int x^n \arg \sec h\frac{x}{a}dx = \frac{x^{n+1}}{n+1}\arg \sec h\frac{x}{a} + \frac{a}{n+1}\int\frac{x^n dx}{\sqrt{a^2-x^2}};$ $\quad (\arg \sec h\frac{x}{a} > 0) =$

$= \frac{x^{n+1}}{n+1}\arg \sec h\frac{x}{a} - \frac{a}{n+1}\int\frac{x^n dx}{\sqrt{a^2-x^2}};$ $\quad (\arg \sec h\frac{x}{a} < 0)$

713. $\int x^n \arg \operatorname{cosec} h\frac{x}{a}dx = \frac{x^{n+1}}{n+1}\arg \operatorname{cosec} h\frac{x}{a} + \frac{a}{n+1}\int\frac{x^n dx}{\sqrt{x^2+a^2}};$ $\quad (x > 0) =$

$= \frac{x^{n+1}}{n+1}\arg \operatorname{cosec} h\frac{x}{a} - \frac{a}{n+1}\int\frac{x^n dx}{\sqrt{x^2+a^2}};$ $\quad (x < 0)$

B$_5$. INTEGRALES DEFINIDAS

a. Definiciones y propiedades

a$_1$. $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

a$_2$. $\int_a^b [k\ f(x)]\,dx = k\int_a^b f(x)\,dx$

a$_3$. $\int_a^a f(x)\,dx = 0$

a$_4$. $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

a_5. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$; (si $f(x)$ es integrable en $[a, b]$ y $a < c < b$)

a_6. Regla de Barrow: $\int_a^b f(x)dx = F(b) - F(a)$;

{si $f(x)$ es continua en $[a, b]$ y $F(x)$ es una primitiva de $f(x)$}

b. Integrales impropias

b_1. $\int_a^{+\infty} f(x)\,dx = \lim_{b \to \infty} \int_a^b f(x)\,dx$ *

b_2. $\int_{-\infty}^b f(x)\,dx = \lim_{a \to -\infty} \int_a^b f(x)\,dx$

b_3. $\int_a^{b^-} f(x)\,dx = \lim_{c + b^-} \int_a^c f(x)\,dx$ *

b_4. $\int_{a^+}^b f(x)\,dx = \lim_{c \to a^+} \int_c^b f(x)$

* El símbolo $\int_a^{+\infty} f(x)\,dx$ se llama integral impropia de primera especie y el símbolo $\int_a^{b^-} f(x)\,dx$ se llama integral impropia de segunda especie.

b_5. $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^0 f(x)\,dx + \int_0^{\infty} f(x)\,dx$; (si $f(x)$ es integrable sobre cada intervalo finito)

c. Integrales definidas o impropias más usuales

c_1. De las funciones racionales e irracionales

714. $\int_0^a \sqrt{a^2 - x^2}\,dx = \frac{\pi a^2}{4}$

715. $\int_0^a \frac{dx}{\sqrt{a^2 - x^2}} = \frac{\pi}{2}$

716. $\int_0^\infty \frac{dx}{x^2 + a^2} = \frac{\pi}{2a}$

717. $\int_0^\infty \frac{x^n dx}{x^m + a^n} = \frac{\pi a^{n+m+1}}{n \ \text{sen}\ (\frac{n+1}{m}\pi)}$; $(0 < n + 1 < m)$

718. $\int_0^a x^n (a^m - x^m)^b dx = \frac{a^{n+m+b+1}}{m} \cdot \frac{\Gamma(\frac{n+1}{m}) \cdot \Gamma(b+1)}{\Gamma(\frac{n+1}{m+b+1})}$; Γ: función gamma

c_2. *De las funciones logarítmicas*

719. $\int_0^1 \frac{\ln x}{1-x} dx = -\frac{\pi^2}{6}$

720. $\int_0^1 \frac{\ln x}{1+x} dx = -\frac{\pi^2}{12}$

721. $\int_0^1 \frac{\ln (1+x)}{x} dx = \frac{\pi^2}{12}$

722. $\int_0^1 \frac{\ln (1-x)}{x} dx = -\frac{\pi^2}{6}$

723. $\int_0^1 x^n \ \ln^m x \ dx = \frac{(-1)^m m!}{(n+1)^{m+1}}$; $(n > -1; m \in No)$

724. $\int_0^1 \frac{\ln x}{x^2 - 1} dx = \frac{\pi^2}{8}$

725. $\int_0^1 \frac{x^n - x^m}{\ln x} dx = \ln \left| \frac{n+1}{m+1} \right|$

726. $\int_0^{\frac{\pi}{2}} \ln \text{sen}\, x \ dx = \int_0^{\frac{\pi}{2}} \ln \cos x \ dx = -\frac{\pi \ln 2}{2}$

727. $\int_0^\pi x \ \ln \text{sen}\, x \ dx = -\frac{\pi^2 \ln^2}{2}$

728. $\int_0^{\frac{\pi}{2}} (\ln \text{sen}\, x)^2 dx = \int_0^{\frac{\pi}{2}} (\ln |\cos x|)^2 dx = \frac{\pi^3}{4!} + \frac{\pi \ln^3 2}{2}$

729. $\int_0^4 \ln\,(1+\operatorname{tg} x)\,dx = \frac{\pi}{8}\ln|2|$

730. $\int_0^\pi \ln\frac{e^x+1}{e^x-1}\,dx = \frac{\pi^2}{4}$

c_3. *De las funciones exponenciales*

731. $\int_0^\infty e^{-ax}\operatorname{sen} bx\;\;dx = \frac{b}{a^2+b^2}$

732. $\int_0^\infty e^{-ax}\cos bx\;\;dx = \frac{a}{a^2+b^2}$

733. $\int_0^\infty \frac{e^{-ax}\operatorname{sen} bx}{x} = \operatorname{arc}\operatorname{tg}\frac{b}{a}$

734. $\int_0^\infty e^{-ax^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{8}}$

735. $\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}\,dx = \ln\left|\frac{b}{a}\right|$

736. $\int_0^\infty x^n e^{-ax}dx = \frac{\Gamma(n+1)}{a^{n+1}}$; Γ: Función gamma

737. $\int_0^\infty \frac{x\;\;dx}{e^{x+1}} = \frac{\pi^2}{12}$

738. $\int_0^\infty \frac{x\;\;dx}{e^x-1} = \frac{\pi^2}{6}$

c_4. *De las funciones trigonométricas*

739. $\int_0^{\frac{\pi}{2}} \operatorname{sen}^2 x\;\;dx = \int_0^{\frac{\pi}{2}} \cos^2 x\;\;dx = \frac{\pi}{4}$

740. $\int_0^\pi \frac{\operatorname{sen} ax}{x}\,dx = \frac{\pi}{2}\ (\text{si } a>0)$ $\qquad = -\frac{\pi}{2}\ (\text{si } a<0)$

741. $\int_0^\infty \frac{\cos ax - \cos bx}{x}\,dx = \ln\left|\frac{b}{a}\right|$

742. $\int_0^\infty \frac{\operatorname{sen} x \ \cos ax}{x} dx = \frac{\pi}{2} \ (\text{si} \, |a| < 1) \quad = \frac{\pi}{4} \ (\text{si} \, |a| = 1) \qquad = 0 \ (\text{si} \, |a| > 1)$

743. $\int_0^\infty \frac{\operatorname{sen} x}{\sqrt{x}} dx = \int_0^\infty \frac{\cos x}{\sqrt{x}} dx = \sqrt{\frac{\pi}{2}}$

744. $\int_0^\infty \frac{\cos ax}{1 + x^2} dx = \frac{\pi}{2} e^{-|a|}$

745. $\int_0^\infty \operatorname{sen} ax^2 dx = \int_0^\infty \cos ax^2 dx = \sqrt{\frac{\pi}{8a}}$

746. $\int_0^\infty \frac{\operatorname{sen}^3 x}{x^3} dx = \frac{3\pi}{8}$

747. $\int_0^\infty \frac{\operatorname{sen}^4 x}{x^4} dx = \frac{\pi}{3}$

748. $\int_0^\infty \frac{\operatorname{tg} x}{x} dx = \frac{\pi}{2}$

749. $\int_0^1 \frac{\operatorname{arc\,sen} x}{x} dx = \frac{\pi}{2} \ln |2|$

750. $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \operatorname{tg}^n x} = \frac{\pi}{4}$

751. $\int_0^\infty \frac{\cos nx}{x^2 + a^2} dx = \frac{\pi}{2a} e^{-na}$

752. $\int_0^\infty \frac{x \ \operatorname{sen} nx}{x^2 + a^2} dx = \frac{\pi}{2} e^{-na}$

753. $\int_0^\infty \frac{\operatorname{sen} nx}{x \, (x^2 + a^2)} dx = \frac{\pi}{2a^2} \, (1 - e^{-na})$

754. $\int_0^{2\pi} \frac{dx}{a + b \ \operatorname{sen} x} = \frac{2\pi}{\sqrt{a^2 - b^2}}$

755. $\int_0^{2\pi} \frac{dx}{a + b \ \cos x} = \frac{\operatorname{arc\,cos} \frac{b}{a}}{\sqrt{a^2 - b^2}}$

756. $\int_0^\infty \frac{sh\ ax}{e^{bx}+1}dx = \frac{x}{2b}\operatorname{cosec}\frac{a\pi}{b} - \frac{1}{2a}$

757. $\int_0^\infty \frac{sh\ ax}{e^{bx}-1} = \frac{1}{2a} - \frac{\pi}{2b}\operatorname{cotg}\frac{a\pi}{b}$

758. $\int_0^\infty \frac{\operatorname{sen} ax}{sh\ bx} = \frac{\pi}{2b}\operatorname{tg} h\frac{a\pi}{2b}$

759. $\int_0^\infty \frac{x\ dx}{sh\ ax} = \frac{\pi^2}{4a^2}$

760. $\int_0^\infty \frac{\cos\ ax}{ch\ bx}dx = \frac{\pi}{2b}\sec h\frac{a\pi}{2b}$