

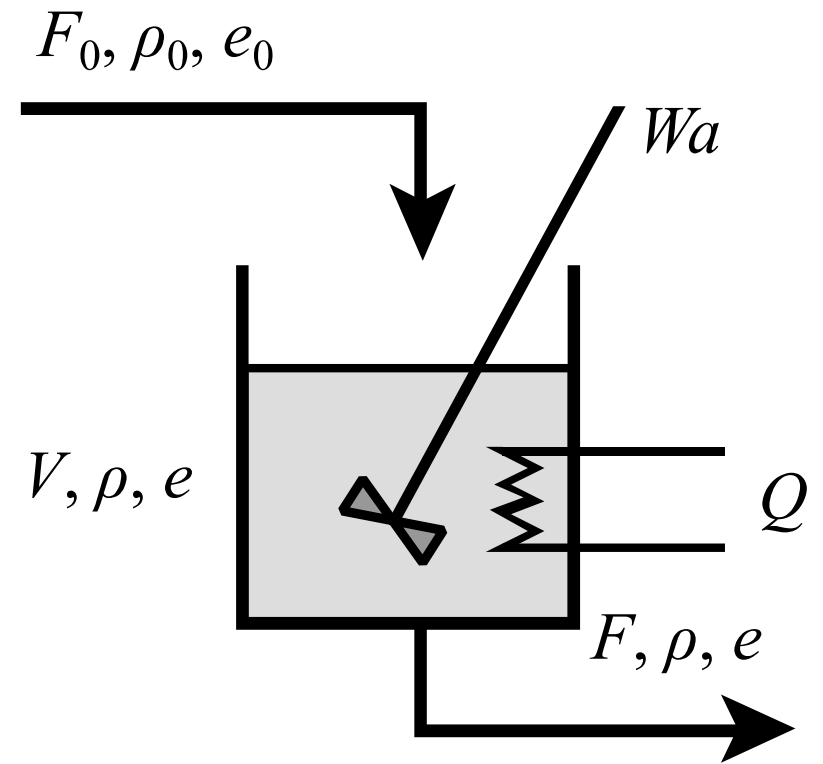
# Fundamentos Parte III

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# Balace de energía

# Balance de energía

- {vel. de acumulación de energía} = {velocidad de entrada de energía} - {velocidad de salida de energía}
- No existe generación.
- [energía]/[tiempo]: J/h
- Un único balance por volumen de control.



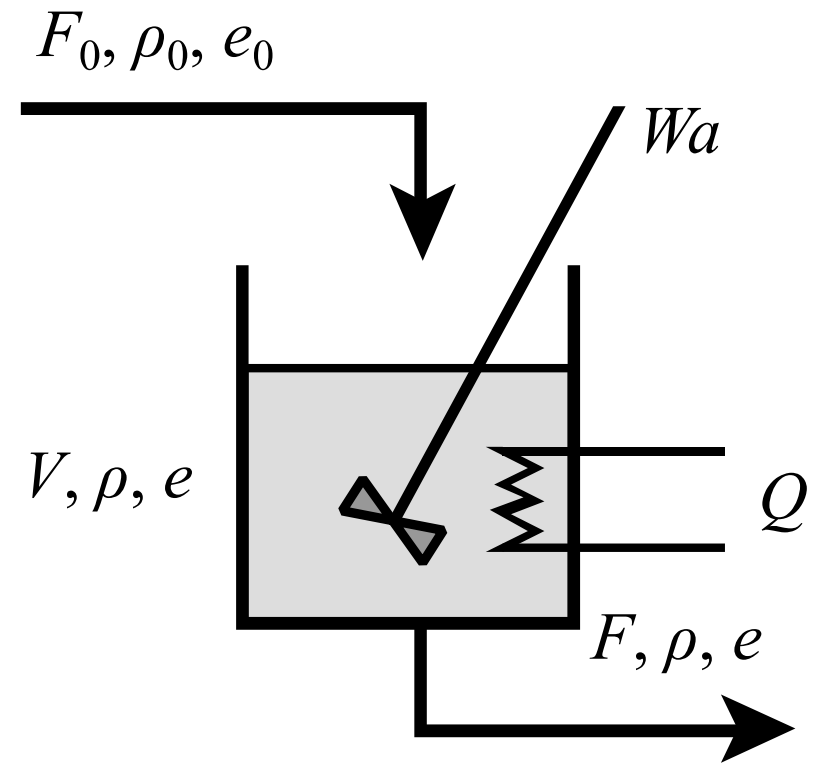
# Balance de energía

- {vel. de acum.}  $= \frac{d(V \rho e)}{dt}$
- {vel. de entrada}  $= F_0 \rho_0 e_0 + Q + Wn$
- {vel. de salida}  $= F \rho e$

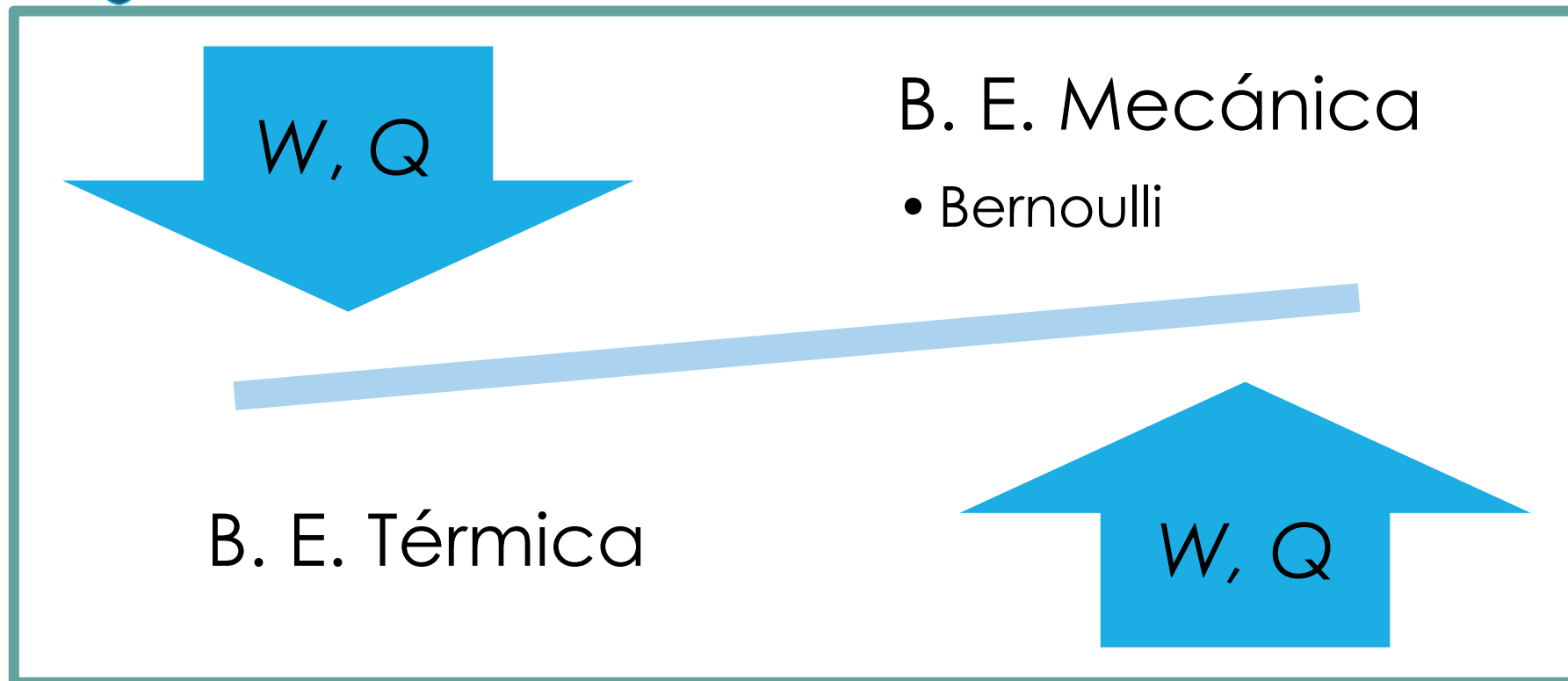
$$\frac{d(V \rho e)}{dt} = F_0 \rho_0 e_0 + Q + Wn - F \rho e$$

$$e = u + \cancel{ek} + ep$$

$$ek = \frac{1}{2} v^2, \quad ep = gy$$



# Energía mecánica y térmica



Balance de energía total

# Balance de energía

$$Wn = F_0 P_0 + Wa - FP$$

$$f_v = P A v = PF$$

$$h = u + \frac{P}{\rho}$$

$$\frac{d(VP)}{dt} = \cancel{V \frac{dP}{dt}} + \cancel{P \frac{dV}{dt}}$$

$$\frac{d(V \rho u)}{dt} = F_0 \rho_0 u_0 + Q + Wn - F \rho u$$

$$\frac{d(V \rho u)}{dt} = F_0 \rho_0 \left( u_0 + \frac{P_0}{\rho_0} \right) + Q + Wa - F \rho \left( u + \frac{P}{\rho} \right)$$

$$\frac{d(V \rho h)}{dt} - \frac{d(VP)}{dt} = F_0 \rho_0 h_0 + Q + Wa - F \rho h$$

$$\frac{d(V \rho h)}{dt} = F_0 \rho_0 h_0 + Q + Wa - F \rho h$$

# Estado de referencia

$$\frac{d(V \rho h)}{dt} = F_0 \rho_0 h_0 + Q + Wa - F \rho h$$

$$V \rho \frac{dh}{dt} + h \frac{d(V \rho)}{dt} = F_0 \rho_0 h_0 + Q + Wa - F \rho h$$

$$-hr \left\{ \frac{d(V \rho)}{dt} = F_0 \rho_0 - F \rho \right\}$$

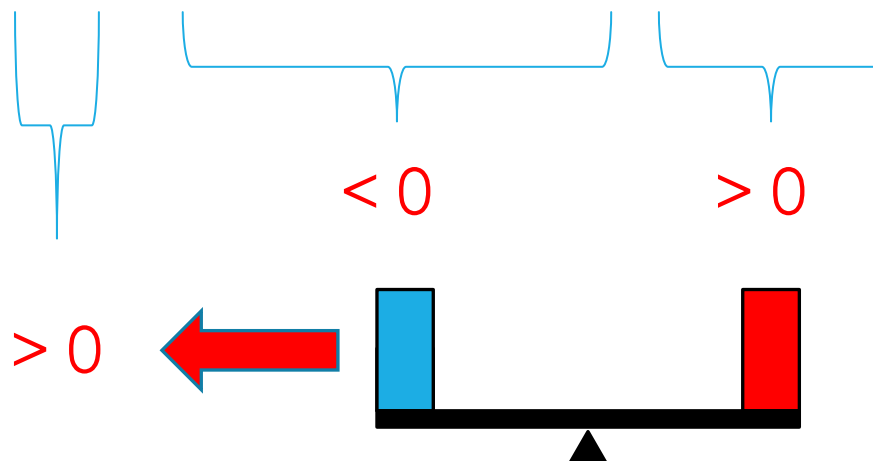
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$$h_1 - h_0 = \int_{T_0}^{T_1} C_p(T) dT \quad V \rho \frac{dh}{dt} + (h - hr) \frac{d(V \rho)}{dt} = F_0 \rho_0 (h_0 - hr) + Q + Wa - F \rho (h - hr)$$

$$hr = h \quad \boxed{V \rho \frac{dh}{dt} = F_0 \rho_0 (h_0 - h) + Q + Wa} \quad \boxed{V \rho C_p \frac{dT}{dt} = F_0 \rho_0 C_{p_0} (T_0 - T) + Q + Wa}$$

# Análisis dinámico cualitativo

- Estado estacionario:
  - $dT/dt = 0$
- Disminuye  $F_0$ :
  - Aumenta  $T(+)$ .

$$V \rho C_p \frac{dT}{dt} = F_0 \rho_0 C_{p_0} (T_0 - T) + Q + W_a = 0$$


The diagram illustrates the dynamic analysis of the equation. It shows a seesaw with a blue block on the left and a red block on the right. A red arrow points left from the blue block, and a red '> 0' is written below it. The red block is higher than the blue block. The equation is annotated with blue brackets: one under  $V \rho C_p \frac{dT}{dt}$ , one under  $F_0 \rho_0 C_{p_0} (T_0 - T)$  with a red '< 0' below it, and one under  $Q + W_a$  with a red '> 0' below it.



# Análisis dinámico cualitativo


- Estado estacionario:
  - $dT/dt = 0$
- Aumenta  $x$ , dado  $F_0 = kx$ :
  - Disminuye  $T(-)$ .

$$V \rho C_p \frac{dT}{dt} = F_0 \rho_0 C_{p_0} (T_0 - T) + Q + Wa = 0$$

The diagram illustrates the signs of the terms in the energy balance equation. The left pan is higher and labeled  $< 0$ , and the right pan is lower and labeled  $> 0$ . A red arrow points left from the pivot, indicating the direction of the net force or the resulting change in temperature.

# Análisis dinámico cualitativo

- Estado estacionario:
  - $dT/dt = 0$
- Disminuye  $F$ , dado  $\frac{dV}{dt} = F_0 - F$ :
  - No cambia  $T(0)$ .

$$V \rho C_p \frac{dT}{dt} = F_0 \rho_0 C_{p_0} (T_0 - T) + Q + W_a = 0$$


The diagram illustrates a balance scale with a blue block on the left and a red block on the right. A red arrow points left from the blue block, and a red arrow points right from the red block. The scale is tilted, with the right side lower. The text  $= 0$  is written to the left of the scale.

# Calor de reacción en un sistema con parámetros concentrados

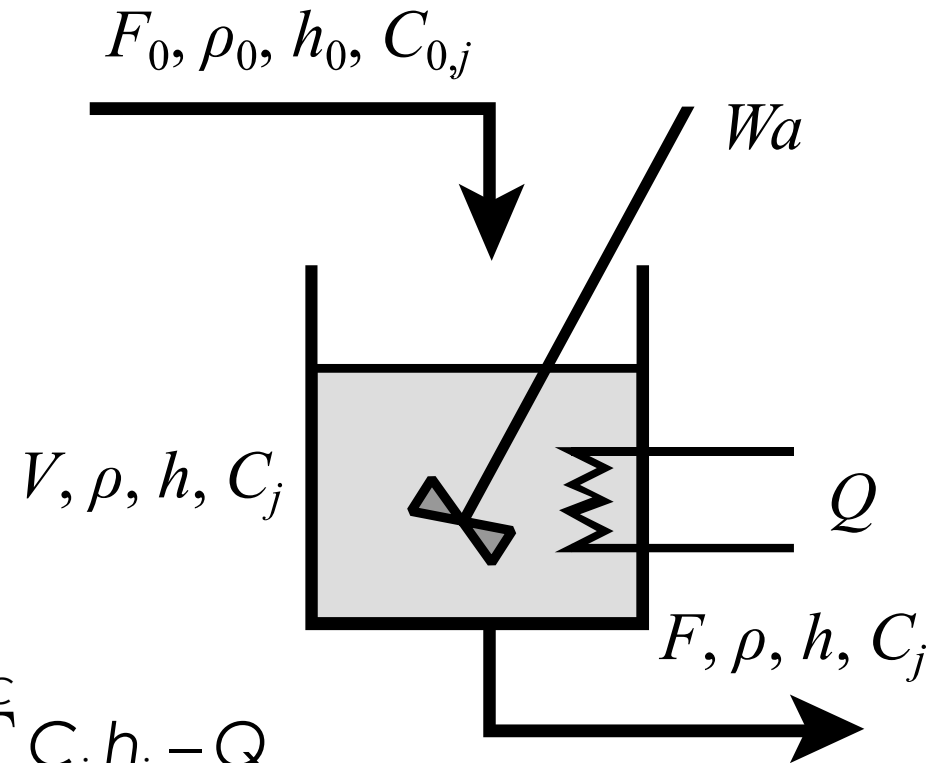
# Solución ideal

- {vel. de acum.}  $= \frac{d}{dt} (V C_A h_A + V C_B h_B)$
- {Vel. de acum.}  $= \frac{d}{dt} (V (C_A h_A + C_B h_B))$
- {Vel. de acum.}  $= \frac{d}{dt} \left( V \sum_{j=1}^{NC} C_j h_j \right)$

# Calor de reacción

- {vel. de acum.} =  $\frac{d}{dt} \left( v \sum_{j=1}^{NC} C_j h_j \right)$
- {vel. de entrada} =  $F_0 \sum_{j=1}^{NC} C_{0,j} h_{0,j} + Wa$
- {vel. de salida} =  $F \sum_{j=1}^{NC} C_j h_j + Q$

$$\frac{d}{dt} \left( v \sum_{j=1}^{NC} C_j h_j \right) = F_0 \sum_{j=1}^{NC} C_{0,j} h_{0,j} + Wa - F \sum_{j=1}^{NC} C_j h_j - Q$$



# Derivada de un producto

$$\begin{aligned} & \frac{d}{dt} \left( v \sum_{j=1}^{NC} C_j h_j \right) \\ &= \frac{d}{dt} \left( \sum_{j=1}^{NC} v C_j h_j \right) \\ &= \sum_{j=1}^{NC} v C_j \frac{dh_j}{dt} + \sum_{j=1}^{NC} h_j \frac{d(v C_j)}{dt} \\ &= v \sum_{j=1}^{NC} C_j \frac{dh_j}{dt} + \sum_{j=1}^{NC} h_j \frac{d(v C_j)}{dt} \end{aligned}$$

$$\begin{aligned} & \frac{d}{dt} (v(C_A h_A + C_B h_B)) \\ &= \frac{d}{dt} (v C_A h_A + v C_B h_B) \\ &= \left( v C_A \frac{dh_A}{dt} + h_A \frac{d(v C_A)}{dt} \right) + \left( v C_B \frac{dh_B}{dt} + h_B \frac{d(v C_B)}{dt} \right) \\ &= v \left( C_A \frac{dh_A}{dt} + C_B \frac{dh_B}{dt} \right) + \left( h_A \frac{d(v C_A)}{dt} + h_B \frac{d(v C_B)}{dt} \right) \end{aligned}$$

# Calor de reacción

$$\frac{d}{dt} \left( V \sum_{j=1}^{NC} C_j h_j \right) = F_0 \sum_{j=1}^{NC} C_{0,j} h_{0,j} + Wa - F \sum_{j=1}^{NC} C_j h_j - Q$$

$$V \sum_{j=1}^{NC} C_j \frac{dh_j}{dt} + \sum_{j=1}^{NC} h_j \frac{d(V C_j)}{dt} = F_0 \sum_{j=1}^{NC} C_{0,j} h_{0,j} + Wa - F \sum_{j=1}^{NC} C_j h_j - Q$$

$$\sum_{j=1}^{NC} h_j \left\{ \frac{d(V C_j)}{dt} = F_0 C_{0,j} + \alpha_j r V - F C_j \right\}$$

$$- \left\{ \sum_{j=1}^{NC} h_j \frac{d(V C_j)}{dt} = F_0 \sum_{j=1}^{NC} h_j C_{0,j} + V r \sum_{j=1}^{NC} \alpha_j h_j - F \sum_{j=1}^{NC} h_j C_j \right\}$$

$$\Delta H = \sum_{j=1}^{NC} \alpha_j h_j$$

$$V \sum_{j=1}^{NC} C_j \frac{dh_j}{dt} = F_0 \sum_{j=1}^{NC} C_{0,j} (h_{0,j} - h_j) + V r (-\Delta H) + Wa - Q$$

$$\Delta H = \Delta H_r + \int_{T_r}^T \Delta C_p(\xi) d\xi \quad \Delta C_p(T) = \sum_{j=1}^{NC} \alpha_j C_{p_j}(T)$$

Ley de Kirchhoff

# Calor de reacción

$$V \sum_{j=1}^{NC} C_j \frac{dh_j}{dt} = F_0 \sum_{j=1}^{NC} C_{0,j} (h_{0,j} - h_j) + Vr(-\Delta H) + Wa - Q$$

$$h_1 - h_0 = \int_{T_0}^{T_1} Cp(T) dT$$

$$V \sum_{j=1}^{NC} C_j Cp_j \frac{dT}{dt} = F_0 \sum_{j=1}^{NC} C_{0,j} Cp_j (T_0 - T) + Vr(-\Delta H) + Wa - Q$$

$$Cp = \sum_{j=1}^{NC} x_j Cp_j$$

$$V C Cp \frac{dT}{dt} = F_0 C_0 Cp_0 (T_0 - T) + Vr(-\Delta H) + Wa - Q$$

Regla de mezcla

$$V \rho Cp \frac{dT}{dt} = F_0 \rho_0 Cp_0 (T_0 - T) + Vr(-\Delta H) + Wa - Q$$

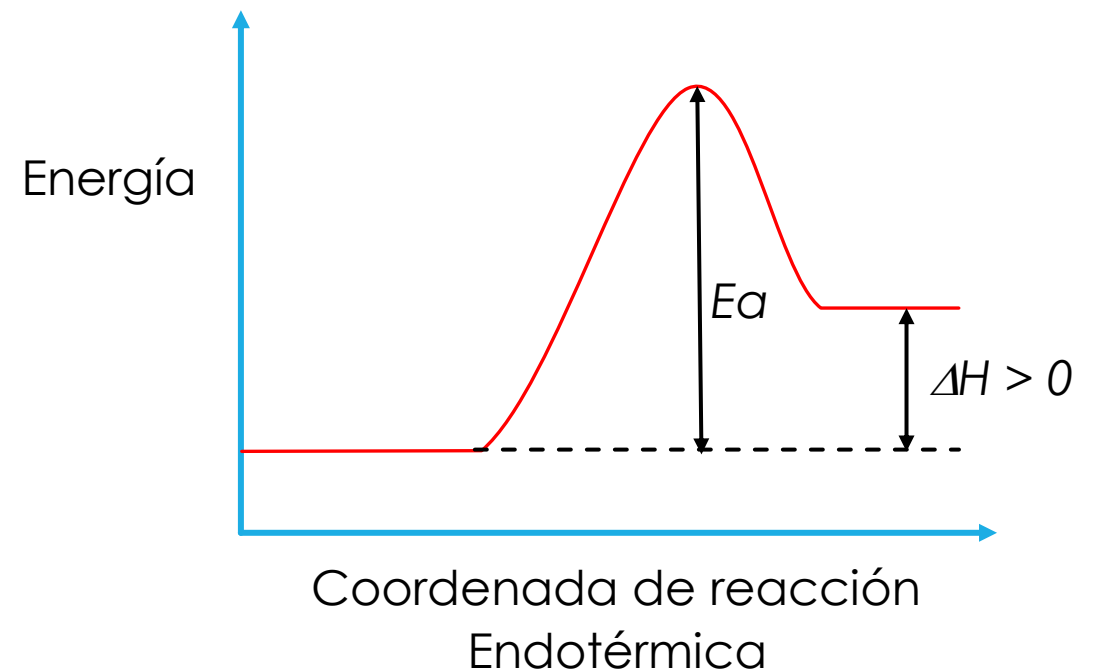
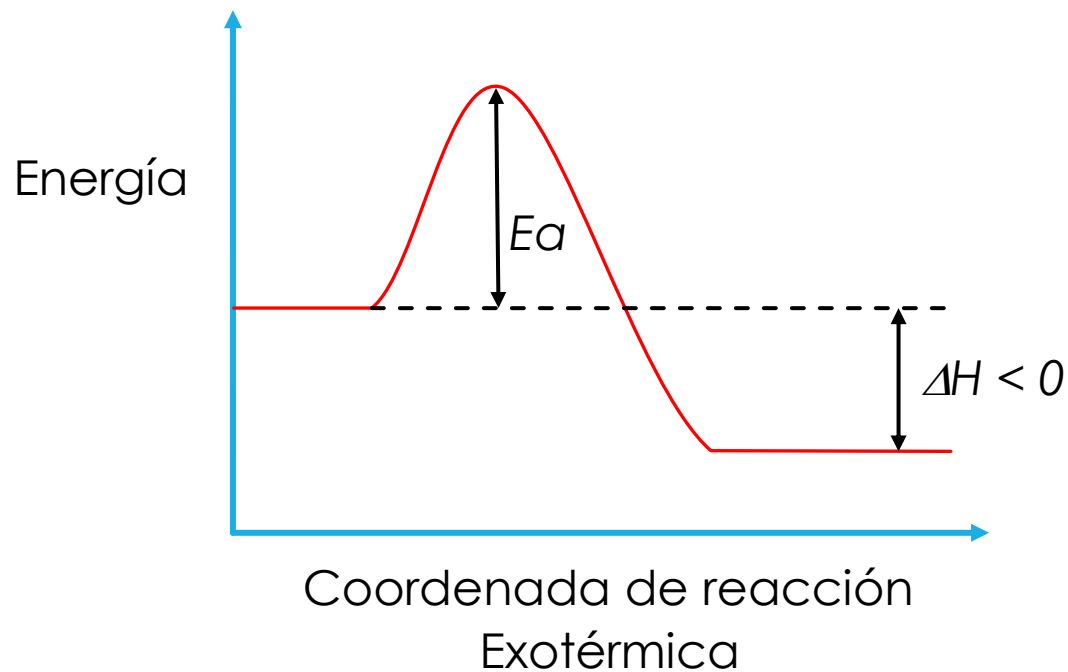
$$C_j = x_j C$$

$$V \rho Cp \frac{dT}{dt} = \sum_{\forall i \in E} F_i \rho_i Cp_i (T_i - T) + V \sum_{\forall k \in R} r_k (-\Delta H_k) + Wa - Q$$



# Calor de reacción

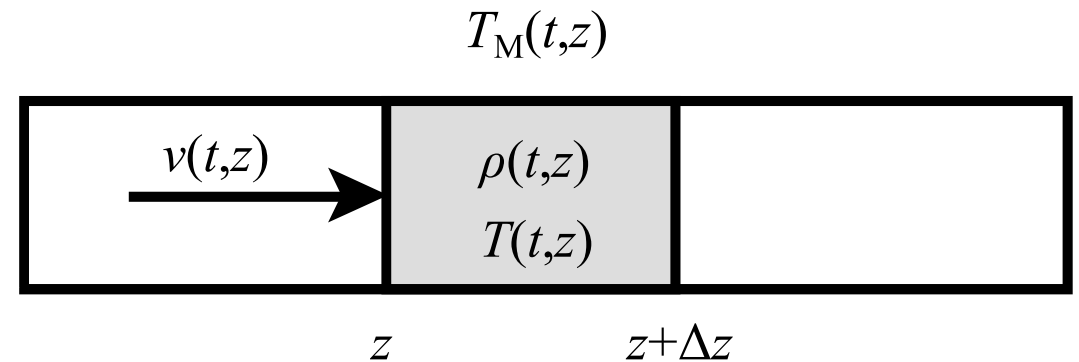
$$V \rho C_p \frac{dT}{dt} = \sum_{\forall i \in E} F_i \rho_i C_{p_i} (T_i - T) + V \sum_{\forall k \in R} r_k (-\Delta H_k) + Wa - Q$$



# Balance de energía en un sistema con parámetros distribuidos

# Balance de energía

- {vel. de acum.} =  $\frac{\partial(A \Delta z \rho h)}{\partial t}$
- {vel. de entrada} =  $v A \rho h|_z + A q|_z + \Delta Q$
- {vel. de salida} =  $v A \rho h|_{z+\Delta z} + A q|_{z+\Delta z}$



$$\frac{\partial(A \Delta z \rho h)}{\partial t} = v A \rho h|_z + A q|_z + \Delta Q - v A \rho h|_{z+\Delta z} - A q|_{z+\Delta z}$$

$$\Delta z \frac{\partial(\rho h)}{\partial t} = v \rho h|_z + q|_z + \frac{1}{A} \Delta Q - v \rho h|_{z+\Delta z} - q|_{z+\Delta z}$$

$$\frac{\partial(\rho h)}{\partial t} = -\frac{v \rho h|_{z+\Delta z} - v \rho h|_z}{\Delta z} - \frac{q|_{z+\Delta z} - q|_z}{\Delta z} + \frac{1}{A} \frac{\Delta Q}{\Delta z}$$

$$\frac{\partial(\rho h)}{\partial t} = -\frac{\partial(v \rho h)}{\partial z} - \frac{\partial q}{\partial z} + \frac{1}{A} \frac{\partial Q}{\partial z} = 0$$

# Variación de la propiedad intensiva

$$\frac{\partial(\rho h)}{\partial t} = -\frac{\partial(v \rho h)}{\partial z} - \frac{\partial q}{\partial z} + \frac{1}{A} \frac{\partial Q}{\partial z}$$

$$\rho \frac{\partial h}{\partial t} + h \frac{\partial \rho}{\partial t} = -h \frac{\partial(v \rho)}{\partial z} - v \rho \frac{\partial h}{\partial z} - \frac{\partial q}{\partial z} + \frac{1}{A} \frac{\partial Q}{\partial z}$$
$$-h \left\{ \frac{\partial \rho}{\partial t} = -\frac{\partial(v \rho)}{\partial z} \right\}$$

$$\rho \frac{\partial h}{\partial t} = -v \rho \frac{\partial h}{\partial z} - \frac{\partial q}{\partial z} + \frac{1}{A} \frac{\partial Q}{\partial z}$$

$$\frac{dh}{dt} = \left( \frac{\partial h}{\partial T} \right)_P \frac{dT}{dt} = C_p \frac{dT}{dt}$$

$$\rho C_p \frac{\partial T}{\partial t} = -v \rho C_p \frac{\partial T}{\partial z} - \frac{\partial q}{\partial z} + \frac{1}{A} \frac{\partial Q}{\partial z}$$