Mining cutoff grade strategy to optimise NPV based on multiyear GRG iterative factor

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One of the most difficult problems in a mining operation is how to determine the optimum minable cutoff grade over the lifespan of the mine that will maximise the operation net present value (NPV). Maximising the NPV in a mining operation, subject to different constraints, is a non-linear programming problem. Cutoff grade optimisation is used to determine the operating mining strategy that will maximise the total profit value of the mine. Constrained by the production capacity of the mill, and by sacrificing the mining low grade material, this approach enables the mill to process ore that delivers an improved time discounted cashflow. The cutoff grade policy calculated from the algorithm introduced in this paper has a significant influence on the overall economics of the mining operation. This paper describes the process to determine the cutoff grade strategy used in a mining operation based on Lane's algorithm using an optimisation factor which is iteratively calculated for every production year which dynamically adjusts the remaining reserves and thus the total life of the mine to maximise the project NPV. The introduced algorithm is an adaptation from Lane's algorithm that incorporates an iterative routine used to calculate the optimisation factor embedded in the cutoff grade equation. The algorithm was developed at Virginia Tech and runs using a windows visual basic program linked to a spreadsheet interface. The benefits of the methodology are demonstrated using a hypothetical case study. The authors have observed an improvement of the total NPV using the general reduced gradient (GRG) approach to iteratively calculate the optimisation factor for every production year.

Keywords: Modelling, Mineral economics, Mining, Reserves, NPV, Cutoff grade optimization

Introduction

For a long time, the 'optimum cutoff grade' has been a controversial concept to metal miners all over the world. In later times, the 'opportunity cost' concept has emerged as a new variable to the definition of 'cutoff grade' when maximisation of the net present value (NPV) is the objective. The typical variables associated with the cutoff grade (COG) definition are the technical mining parameters such as recovery, capacities, etc. and a given economic scenario such as metal prices, mining and processing costs, etc.

New research is being focused on developing new optimisation techniques based on Lane's algorithm to determine the COG policy.[1] Cutoff grade optimisation is used to derive an operating strategy that maximises the profit of a mine. This profit maximisation is based on sacrificing low grade material during the first stage of the mine. This will send high grade ore to the mill resulting in a higher cashflow. Therefore, the COG policy has a significant influence on the overall economics of the mining operation. The determination of the COG policy, which maximises the NPV, has already been established in the industry. It is proved that as opposed to a constant breakeven COG, variable COGs – which change owing to the declining effect of NPV during the life of the mine – not only honor the metal price and cash costs of mining, milling and refining stages, but also take into account the limiting capacities of these stages and the grade–tonnage distribution of the deposit.[1,2] In other words, the techniques that determine the optimum COG policy consider the opportunity cost of not receiving the future cash flows earlier during the mine life owing to the limiting capacities present in the stages of mining, milling, or refining .[3]

Optimisation of COG

Optimising mine COGs have long been searched as the means to maximise the NPV of mining projects. In recent years, the theory behind mine COG optimisation has been extended to encompass other parts of the operation. Primarily, this involves optimising the throughput and recovery in the mill. A mill may be operated at a high throughput rate, sacrificing recovery,

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but achieving a higher rate of production of mineral. This is equivalent to a mine operating at an above breakeven COG, sacrificing some of the low grade resource to obtain higher cash flow. This analogy suggests that tools developed for determining the optimum COG strategy in the mine might also be used to find a solution for the mill. Because the mine and mill are interdependent, it is not feasible to optimise the process by treating both independently.

Cutoff grade optimisation maximises the NPV of a project subject to capacity constraints in the mine, mill and the economic market. These are usually expressed as annual constraints to the tonnage mined, tonnage milled and product sold. At any given point in time at least one constraint, and possibly two, will constrain the system. For cutoff optimisation to work correctly, capacity constraints must be independent of the COG.[4]

Lane has developed a comprehensive approach to determine the COG policy.[1,3] Whittle and Wharton enriched the approach by including the opportunity cost variable into the equation. This concept is referred to as delay cost and the change cost.[5] Asad, in contrast, used the dynamic metal price and cost escalation during mine life for COG optimisation.[6] Table 1 shows the notation used in Lane's algorithm.

In his approach, Lane demonstrates that a COG calculation which maximises NPV has to include the fixed cost associated with not receiving the future cash flows quicker owing to the COG decision taken now. The COG, when the concentrator is the constraint, is given below.

$$g_{\rm m}(i)=\frac{c+f+F_{\rm i}}{(P-s)y} \qquad (1)$$

where $g_{\rm m}(i)$ is the milling COG, f is the fixed cost, $F_{\rm i}$ is the opportunity cost per ton of material milled in year i, P is the profit (\$) per unit sold, s is the selling price per commodity unit (\$/unit of product), y is the mill recovery (%).

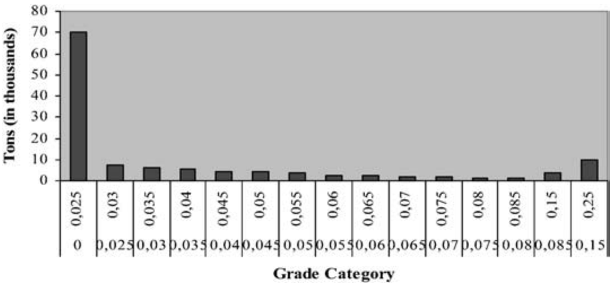
The opportunity cost is determined as

$$F_{\rm i}=\frac{dNPV_{\rm i}}{C} \qquad (2)$$

where d is the discount rate, $NPV_{\rm i}$ is the NPV of the future cash flows of the years i to the end mine life N, and the C is the total milling capacity in year i.

Table 1 Notations of algorithm

Notation	Explain	Unit
i	Year	–
N	Mine life	year
P	Metal price	\$/oz
s	Sales cost	\$/oz
m	Mining cost	\$/t
c	Processing cost	\$/t
$f_{\rm a}$	Fixed costs	\$/year
f	Fixed cost	\$/ton
y	Recovery	%
d	Discount rate	%
CC	Capital costs	\$
M	Mining capacity	t/year
C	Milling capacity	t/year
R	Refining capacity	t/year
$Q_{\rm m}$	Material mined	t/year
$Q_{\rm c}$	Ore processed	t/year
$Q_{\rm r}$	Concentrate refined	t/year



1 Grade–tonnage distribution of deposit

$$f=\frac{f_{\rm a}}{C} \qquad (3)$$

where $f_{\rm a}$ is the annual fixed costs.

In this study, the algorithm based on Lane's approach was written in VISUAL BASIC using a spreadsheet serving as the user interface resulting in a user friendly windows interface. In addition, the introduced algorithm has been updated with an optimisation factor $\sigma_{\rm t}$ substituting f and $F_{\rm i}$ in the Lane's equation. The optimisation factor is iteratively calculated by the program using GRG iterations. As shown later in this paper, the total NPV in the course of the mine life was improved using this new approach. The computer program incorporates the optimisation factor $\sigma_{\rm t}$ which is included in the ultimate COG equation, which considers the processing cost c and the mining cost m per ton, as indicated in equation (4).

$$g_{\rm u}(i)=\frac{c+m+\sigma_{\rm t}}{(P-s)y} \qquad (4)$$

where σ is the optimisation factor. This approach further maximises the total NPV of the mine project.

Probably, the most important role of this approach is that it calculates the optimisation factor $\sigma_{\rm t}$ in an iterative approach updating the remaining reserves, thus the mine life, at every year, in each iteration, in order to maximise the NPV of the project. This new approach using a variable optimisation factor in a year by year basis resulted in an improved total NPV as shown later in this paper. The program solves for the optimisation factor σ by maximising the project NPV which is based on the ore tonnage–grade distribution and economic parameters of the mine (*see* Table 2 and Fig. 1). The COG mathematical model resulted in a non-linear problem (NLP) which is solved using an iterative process based on the generalised reduced gradient (GRG) algorithm. The GRG method is a powerful computer algorithm used to solve NLPs and is integrated to the program code as a subroutine running within the spreadsheet code module. The GRG algorithm was first developed by Ladson *et al.*[7] and it is one of the techniques that fall within the reduced gradient or gradient projection categories used in mathematics and computer sciences to solve non-linear models.

The COG policy calculated, dictates the quantity to be mined, processed and refined in a given period '*i*', and accordingly, the profits become dependent on the definition of COG. Therefore, the solution of the problem is in the determination of an optimum COG in a given period, which ultimately maximises the objective function.[6]

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Algorithm procedure of COG optimisation

The COG $g_m(i)$ depends on the NPV_i, and the NPV_i cannot be determined until the optimum COGs have been found. The solution to this type of interdependency problem is obtained by an iterative approach as can be seen in the following description of algorithm used.

The steps of the algorithm as mentioned above (equation (1)) are as following:

(i) read the input files: economic parameters (price, selling cost, capacities, etc.) and grade–tonnage distribution

2 (ii) determine the COG $g_m(i)$ by equation (1); setting $V=NPV_i$, the initial $NPV_i=0$

(iii) compute the ore tonnage T_o and waste tonnage T_w from the grade–tonnage curve of the deposit: the ore tonnage T_o and the grade g_c above the COG $g_m(i)$; the waste tonnage T_w that is below the COG $g_m(i)$ and also, compute the stripping ratio SR where $SR=T_w/T_o$

(iv) set $Q_{ci}=C$, if T_o is greater than the milling capacity, otherwise, $Q_{ci}=T_o$. Also, set the Q_{mi} quantity mined ($Q_{mi}=Q_c(1+SR)$) and $Q_{ri}=Q_c g_{avg} y$

(v) determine the annual profit by the following equation

$$P_i=(S_i-r_i)Q_{ri}-Q_{ci}(c_i+f)-m_i Q_{mi}$$

(vi) adjusting the grade–tonnage curve of the deposit by subtracting ore tons Q_{ci} from the grade distribution intervals above optimum COG $g_m(i)$ and the waste tons $Q_{mi}-Q_{ci}$ from the intervals below optimum COG $g_m(i)$ in proportionate among such that the grade–tonnage distribution is not changed.

(vii) check, if Q_{ci} is less than the milling capacity C, then set mine life $N=i$ and go to step (viii); otherwise set the year indicator $i=i+1$ and go to step (ii).

(viii) calculate the accumulated future NPV_i's based on the profits P_i calculated in step (v) for each year from i to N by the following equation $NPV_i=\sum_{j=1}^{N}\frac{P_j}{(1+d)^{j-i+1}}$ for each year $i=1$, N where N is total mine life in years.

Total NPV is calculated at this stage considering a optimisation factor equal to zero. $g_u(i)=\frac{c+m+\sigma_t}{(P-s)y}$ where σ is the optimisation factor equal to zero (*see* Table 2).

(ix) the first iteration is performed by adjusting the optimisation factor σ using the GRG method maximising the NPV_i computed in step (viii). If the computed NPV_i does not converge, the algorithm goes to step (ii) and calculates the COG by varying the embedded optimisation factor σ. If the computed NPV converges, the program stops. The COG g_i's for years $i=1$, N is

Table 2 Cutoff grades calculated before iterative calculated optimisation factor*

		Total NPV		Calculated by program	
		$163 145 601·77			
Year	Profit per year	Cumulative of future NPVs	NPV discounted per year	Optimisation factor	Ultimate COG
0	20 312 990	163 145 602	20 312 990	–	0·039
1	20 312 990	142 832 612	17 818 412	–	0·039
2	20 312 990	125 014 199	15 630 186	–	0·039
3	20 312 990	109 384 013	13 710 690	–	0·039
4	20 312 990	95 673 323	12 026 921	–	0·039
5	20 312 990	83 646 402	10 549 931	–	0·039
6	20 312 990	73 096 472	9 254 325	–	0·039
7	20 312 990	63 842 147	8 117 829	–	0·039
8	20 312 990	55 724 318	7 120 903	–	0·039
9	20 312 990	48 603 415	6 246 406	–	0·039
10	20 312 990	42 357 009	5 479 303	–	0·039
11	20 312 990	36 877 706	4 806 406	–	0·039
12	20 312 990	32 071 300	4 216 146	–	0·039
13	20 312 990	27 855 154	3 698 374	–	0·039
14	20 312 990	24 156 780	3 244 187	–	0·039
15	20 312 990	20 912 593	2 845 778	–	0·039
16	20 312 990	18 066 814	2 496 297	–	0·039
17	20 312 990	15 570 517	2 189 734	–	0·039
18	20 312 990	13 380 783	1 920 819	–	0·039
19	20 312 990	11 459 964	1 684 929	–	0·039
20	20 312 990	9 775 034	1 478 008	–	0·039
21	20 312 990	8 297 026	1 296 498	–	0·039
22	20 312 990	7 000 528	1 137 279	–	0·039
23	20 312 990	5 863 249	997 613	–	0·039
24	20 312 990	4 865 635	875 099	–	0·039
25	20 312 990	3 990 536	767 631	–	0·039
26	20 312 990	3 222 905	673 361	–	0·039
27	20 312 990	2 549 544	590 667	–	0·039
28	20 312 990	1 958 877	518 129	–	0·039
29	20 312 990	1 440 748	454 499	–	0·039
30	20 312 990	986 248	398 684	–	0·039
31	20 312 990	587 565	349 722	–	0·039
32	15 748 691	237 842	237 842	–	0·039
33	–	–	–	–	0·039

*Optimisation factor is zero and total discounted NPV is $163 million.

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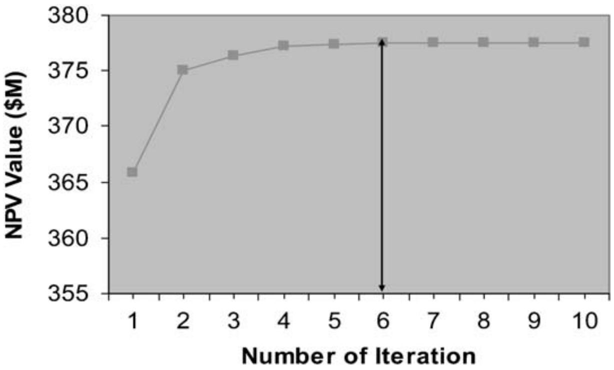

2 Changes of NPV *v.* number of iteration

the optimum policy that maximises the NPV. Table 3 shows the calculated optimisation factors and COG calculated for each year.

Net present value changes per iteration are graphed in Fig. 2 to show the converging behaviour. For example, from the first iteration to the second iteration the NPV changed from $365 801 976 to $375 056 288. Several iterations are performed by the program in order to maximise the NPV. In this particular case after the sixth iteration the NPV reaches its maximum. Also, note the dynamic approach used in every year to calculate the available reserves and thus the new mine life. This process is performed in every iteration until the program converges into a maximum NPV. As described, the iterative optimisation factor approach is used to maximise the NPV then the results are compared with traditional breakeven COG estimation techniques, and they are compared with results coming from COG estimations based on Lane's algorithm. Final results are presented next.

Algorithm application and results

Consider the following hypothetical case of an open pit gold mine.[2] The tonnage–grade distribution and mine design parameters are shown in Table 4 and Fig. 1 respectively. The values in Table 4 give assumed capacities and accepted costs to mine this deposit at

Table 4 Mine design parameters

Parameter	Value
Price, $/oz	500
Sales cost, $/oz	4
Processing cost, $/t	17
Mining cost, $/t	1·3
Capital costs, $	154 Million
Fixed costs f_a, $/year	9·2 Million
Fixed cost f, $/t	9·2
Mining capacity	–
Milling capacity	1·00 Million
Discount rate, %	14
Recovery, %	95

Table 3 Calculated optimisation factors and COGs after GRG iteration process*

		Total NPV			
		$ 373 123 720·52		Calculated by program	
Year	Profit per year	Cumulative of future NPVs	NPV discounted per year	Optimisation factor	Ultimate COG
0	64 477 815	373 123 721	64 477 815	28·72	0·100
1	63 741 174	308 645 906	55 913 311	28·33	0·099
2	60 766 857	252 732 595	46 758 124	26·63	0·095
3	56 799 484	205 974 471	38 338 034	23·96	0·090
4	64 558 686	167 636 438	38 223 925	29·58	0·102
5	64 215 086	129 412 513	33 351 304	31·95	0·107
6	64 652 094	96 061 209	29 454 624	28·91	0·100
7	64 384 047	66 606 585	25 730 268	30·79	0·104
8	53 065 655	40 876 317	18 602 646	20·93	0·083
9	41 080 181	22 273 671	12 632 482	12·68	0·066
10	35 742 022	9 641 189	9 641 189	9·15	0·058

*In this example NPV is maximised from $163 million to $377 million; total mine life is reduced from 32 to 10 years.

Table 5 Breakeven COG policy of gold mine

Year	Optimum cutoff grade, oz/t	Quantity mined, t	Quantity concentrated, t	Quantity refined, t	Profit, $
1	0·039	3 333 869	1 000 000	93 468	15 826 346
2	0·039	3 333 869	1 000 000	93 468	15 826 346
3	0·039	3 333 869	1 000 000	93 468	15 826 346
4	0·039	3 333 869	1 000 000	93 468	15 826 346
5	0·039	3 333 869	1 000 000	93 468	15 826 346
6	0·039	3 333 869	1 000 000	93 468	15 826 346
7	0·039	3 333 869	1 000 000	93 468	15 826 346
8	0·039	3 333 869	1 000 000	93 468	15 826 346
9	0·039	3 333 869	1 000 000	93 468	15 826 346
10–38	0·039	3 333 869	1 000 000	93 468	15 826 346
Total		126 687 022	38 000 000	3 551 784	601 401 148 NPV ($127 984 981)

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2857·14 t/day milling rate. The mine will operate three shifts, seven days per week, 350 days per year.

The traditional COG is defined as breakeven COG and referred to as the ultimate pit COG

$$\text{Ultimate pit cutoff grade} = \frac{\text{Mining cost} + \text{Milling cost}}{(\text{Price} - \text{Cost of sales}) \times \text{Recovery}} \quad (5)$$

Table 5 presents the breakeven COG. As indicated in Table 5, this approach gives a total NPV of \$128·0 million and \$601·4 million of undiscounted profit.

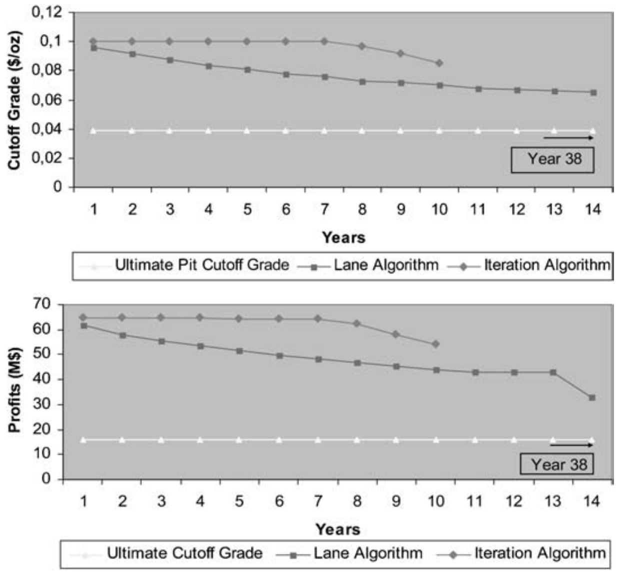
Table 6 presents the optimum COG policy based on Lane's algorithm using the fixed cost and opportunity cost:

$$\frac{m + c + f + F}{(P - s)r} \quad (6)$$

3 As indicated in Table 6, this optimisation approach gives a total NPV of \$354·6 million and \$676·5 million of none discounted profit.

On the other hand, Table 7 presents the results of the optimum COG approach using the optimisation factor σ_t for year by year. The COG policy that is determined by this optimising approach using the optimisation factor σ_t gives a total NPV of \$377·5 million and \$676·5 million of none discounted profit.

According to the values shown in Tables 5–7, the COG policy determined by the optimisation approach using the σ_t factor which is determined year by year (Table 7) gives a higher NPV than the COG policy estimated using the traditional breakeven and Lane's COG approach. The results are given graphically in Fig. 3.



3 Comparison of traditional cutoff grade, K. Lane's and iteration algorithm

As can be seen in Fig. 2, the program stops after the sixth iteration achieving a total NPV of \$377 476 180. After the sixth iteration the optimisation factor converges and the NPV value remains constant.

Table 6 Cutoff grade policy of gold mine using Lane's algorithm

Year	Optimum cutoff grade, oz/t	Quantity mined, t	Quantity concentrated, t	Quantity refined, t	Profit, \$	NPV, \$
1	0·096	12 274 288	1 000 000	209 158	61 585 935	354 674 647
2	0·092	11 336 481	1 000 000	199 710	58 118 871	293 088 712
3	0·088	10 624 419	1 000 000	192 537	55 486·385	242 107 246
4	0·084	10 069 606	1 000 000	186 947	53 435 245	199 412 274
5	0·081	9 628 546	1 000 000	182 504	51 804 650	163 345 006
6	0·078	9 182 735	1 000 000	177 583	49 943 365	132 672 494
7	0·076	8 797 020	1 000 000	173 216	48 278 930	106 733 475
8	0·073	8 437 322	1 000 000	168 992	46 651 502	84 738 244
9	0·072	8 132 220	1 000 000	165 372	45 252 454	66 094 563
10	0·070	7 880 241	1 000 000	162 369	44 090 812	50 230 905
11	0·068	7 652 378	1 000 000	159 528	42 977 995	36 672 630
12	0·067	7 652 378	1 000 000	159 528	42 977 995	25 079 582
13	0·066	7 652 378	1 000 000	159 528	42 977 995	14. 910 242
14	0·065	7 652 378	809 551	129 146	32 898 270	5 989 768
Total		126 972 390	13 809 551	2 426 118	\$676 480 404	\$354 674 647

Table 7 Cutoff grade policy of gold mine using optimisation factor σ_t 4

Year	Optimisation factor σ_t	Optimum cutoff grade, oz/t	Quantity mined, t	Quantity concentrated, t	Quantity refined, t	Profit, \$	NPV, \$
1	28·87	0·100	13 112 225	1 000 000	217 550	4 658 907	377 476 180
2	28·82	0·100	13 107 389	1 000 000	217 550	4 665 194	312 817 273
3	28·83	0·100	13 108 385	1 000 000	217 550	4 663 899	256 093 418
4	28·83	0·100	13 108 178	1 000 000	217 550	4 664 169	206 336 648
5	28·68	0·100	13 034 557	1 000 000	216 818	4 396 643	162 690 175
6	28·65	0·100	13 018 831	1 000 000	216 659	4 338 503	124 562 193
7	28·60	0·100	12 994 065	1 000 000	216 410	4 246 945	91 146 790
8	27·50	0·097	12 450 271	1 000 000	210 931	2 236 542	61 876 747
9	24·90	0·092	11 335 723	1 000 000	199 703	8 116 070	37 004 702
10	21·81	0·085	10 245 376	1 000 000	188 718	4 085 064	16 631 587
Total			125 515 000	10 000 000	2 119 439	626 071 936	377 476 180

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Conclusions

Net present value calculated from the introduced case study indicates that the optimisation factor σ_t on the objective function (NPV) is significant maximising the original NPV $127·99 million, 38 years mine life to $377·5 million, 10 years mine life an equivalent 195% improve. If compared with Lane's approach, the introduced algorithm improves the total NPV by 6·4% from $354·7 million, 14 years mine life.

The COG optimisation algorithm presented here is a new alternative approach that can improve the COG policy of a mine. As currently under research in Virginia Tech, this program could serve as a user friendly template to optimise the COG policy based on grade–tonnage distributions linked to the real geometry of the ore block model and NPV optimisation which is based on multiple ore source grades. This algorithm could also serve to incorporate variable production costs based on stripping ratios interactively calculated from the ore block model.

This approach provides user friendly flexibility for evaluation of various economic case scenarios during the feasibility and planning stages. The program has been developed within a windows environment and is available at the Mining and Minerals Engineering Department, Virginia Tech.

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