Parte B

TABLAS

Sección I. Integrales

TABLA 1 TABLA DE INTEGRALES INDEFINIDAS

En estas tablas de integrales indefinidas (primitivas), a, b, p, q, n son constantes, restringidas si así se especifica; $e = 2{,}71828\ldots$ es la base de los logaritmos naturales. $\ln u$ denota el logaritmo natural de u, donde se sobreentiende que $u > 0$ [en general, para extender la validez de una fórmula al caso $u < 0$ basta sustituir $\ln u$ por $\ln |u|$]. Todos los ángulos están medidos en radianes. Se han omitido todas las constantes de integración. Se supone además que la división por cero está excluida.

Hemos clasificado las integrales por tipos, según cuáles de estas expresiones o funciones están contenidas en el integrando:

(1) $ax + b$
(2) $\sqrt{ax + b}$
(3) $ax + b$ y $px + q$
(4) $\sqrt{ax + b}$ y $px + q$
(5) $\sqrt{ax + b}$ y $\sqrt{px + q}$
(6) $x^2 + a^2$
(7) $x^2 - a^2$, con $x^2 > a^2$
(8) $a^2 - x^2$, con $x^2 < a^2$
(9) $\sqrt{x^2 + a^2}$
(10) $\sqrt{x^2 - a^2}$
(11) $\sqrt{a^2 - x^2}$
(12) $ax^2 + bx + c$
(13) $\sqrt{ax^2 + bx + c}$
(14) $x^3 + a^3$
(15) $x^4 \pm a^4$
(16) $x^n \pm a^n$
(17) $\operatorname{sen} ax$
(18) $\cos ax$
(19) $\operatorname{sen} ax$ y $\cos ax$
(20) $\operatorname{tg} ax$
(21) $\cot ax$
(22) $\sec ax$
(23) $\csc ax$
(24) funciones trigonométricas inversas
(25) e^{ax}
(26) $\ln x$
(27) $\operatorname{sh} ax$
(28) $\operatorname{ch} ax$
(29) $\operatorname{sh} ax$ y $\operatorname{ch} ax$
(30) $\operatorname{th} ax$
(31) $\coth ax$
(32) $\operatorname{sech} ax$
(33) $\operatorname{csch} ax$
(34) funciones hiperbólicas inversas

En algunas integrales aparecen los números B_n de Bernoulli y los E_n de Euler, definidos en las páginas 135-136.

(1) INTEGRALES QUE CONTIENEN $ax + b$

1.1.1 $$\int \frac{dx}{ax + b} = \frac{1}{a} \ln (ax + b)$$

1.1.2 $$\int \frac{x\,dx}{ax + b} = \frac{x}{a} - \frac{b}{a^2} \ln (ax + b)$$

1.1.3 $$\int \frac{x^2\,dx}{ax + b} = \frac{(ax + b)^2}{2a^3} - \frac{2b(ax + b)}{a^3} + \frac{b^2}{a^3} \ln (ax + b)$$

1.1.4 $$\int \frac{dx}{x(ax + b)} = \frac{1}{b} \ln \left(\frac{x}{ax + b}\right)$$

1.1.5 $$\int \frac{dx}{x^2(ax + b)} = -\frac{1}{bx} + \frac{a}{b^2} \ln \left(\frac{ax + b}{x}\right)$$

1.1.6 $$\int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)}$$

1.1.7 $$\int \frac{x\,dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln(ax+b)$$

1.1.8 $$\int \frac{x^2\,dx}{(ax+b)^2} = \frac{ax+b}{a^3} - \frac{b^2}{a^3(ax+b)} - \frac{2b}{a^3}\ln(ax+b)$$

1.1.9 $$\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} + \frac{1}{b^2}\ln\left(\frac{x}{ax+b}\right)$$

1.1.10 $$\int \frac{dx}{x^2(ax+b)^2} = \frac{-a}{b^2(ax+b)} - \frac{1}{b^2x} + \frac{2a}{b^3}\ln\left(\frac{ax+b}{x}\right)$$

1.1.11 $$\int \frac{dx}{(ax+b)^3} = \frac{-1}{2(ax+b)^2}$$

1.1.12 $$\int \frac{x\,dx}{(ax+b)^3} = \frac{-1}{a^2(ax+b)} + \frac{b}{2a^2(ax+b)^2}$$

1.1.13 $$\int \frac{x^2\,dx}{(ax+b)^3} = \frac{2b}{a^3(ax+b)} - \frac{b^2}{2a^3(ax+b)^2} + \frac{1}{a^3}\ln(ax+b)$$

1.1.14 $$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{(n+1)a}. \quad \text{Si } n=-1, \text{ véase 1.1.1.}$$

1.1.15 $$\int x(ax+b)^n\,dx = \frac{(ax+b)^{n+2}}{(n+2)a^2} - \frac{b(ax+b)^{n+1}}{(n+1)a^2}, \quad n \neq -1, -2$$

Si $n = -1, -2$, véanse 1.1.2 y 1.1.7.

1.1.16 $$\int x^2(ax+b)^n\,dx = \frac{(ax+b)^{n+3}}{(n+3)a^3} - \frac{2b(ax+b)^{n+2}}{(n+2)a^3} + \frac{b^2(ax+b)^{n+1}}{(n+1)a^3}$$

Si $n = -1, -2, -3$, véanse 1.1.3, 1.1.8 y 1.1.13.

1.1.17 $$\int x^m(ax+b)^n\,dx = \begin{cases} \dfrac{x^{m+1}(ax+b)^n}{m+n+1} + \dfrac{nb}{m+n+1}\displaystyle\int x^m(ax+b)^{n-1}\,dx \\[2ex] \dfrac{x^m(ax+b)^{n+1}}{(m+n+1)a} - \dfrac{mb}{(m+n+1)a}\displaystyle\int x^{m-1}(ax+b)^n\,dx \\[2ex] \dfrac{-x^{m+1}(ax+b)^{n+1}}{(n+1)b} + \dfrac{m+n+2}{(n+1)b}\displaystyle\int x^m(ax+b)^{n+1}\,dx \end{cases}$$

(2) INTEGRALES QUE CONTIENEN $\sqrt{ax+b}$

1.2.1 $$\int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a}$$

1.2.2 $$\int \frac{x\,dx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b}$$

1.2.3 $$\int \frac{x^2\,dx}{\sqrt{ax+b}} = \frac{2(3a^2x^2-4abx+8b^2)}{15a^3}\sqrt{ax+b}$$

1.2.4 $$\int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \dfrac{1}{\sqrt{b}} \ln\left(\dfrac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right) \\ \dfrac{2}{\sqrt{-b}} \operatorname{tg}^{-1}\sqrt{\dfrac{ax+b}{-b}} \end{cases}$$

1.2.5 $$\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}}$$ [Véase 1.2.12].

1.2.6 $$\int \sqrt{ax+b}\,dx = \frac{2\sqrt{(ax+b)^3}}{3a}$$

1.2.7 $$\int x\sqrt{ax+b}\,dx = \frac{2(3ax-2b)}{15a^2}\sqrt{(ax+b)^3}$$

1.2.8 $$\int x^2\sqrt{ax+b}\,dx = \frac{2(15a^2x^2-12abx+8b^2)}{105a^3}\sqrt{(ax+b)^3}$$

1.2.9 $$\int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}$$ [Véase 1.2.12].

1.2.10 $$\int \frac{\sqrt{ax+b}}{x^2}\,dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{ax+b}}$$ [Véase 1.2.12].

1.2.11 $$\int \frac{x^m}{\sqrt{ax+b}}\,dx = \frac{2x^m\sqrt{ax+b}}{(2m+1)a} - \frac{2mb}{(2m+1)a}\int \frac{x^{m-1}}{\sqrt{ax+b}}\,dx$$

1.2.12 $$\int \frac{dx}{x^m\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(m-1)bx^{m-1}} - \frac{(2m-3)a}{(2m-2)b}\int \frac{dx}{x^{m-1}\sqrt{ax+b}}$$

1.2.13 $$\int x^m\sqrt{ax+b}\,dx = \frac{2x^m}{(2m+3)a}(ax+b)^{3/2} - \frac{2mb}{(2m+3)a}\int x^{m-1}\sqrt{ax+b}\,dx$$

1.2.14 $$\int \frac{\sqrt{ax+b}}{x^m}\,dx = -\frac{\sqrt{ax+b}}{(m-1)x^{m-1}} + \frac{a}{2(m-1)}\int \frac{dx}{x^{m-1}\sqrt{ax+b}}$$

1.2.15 $$\int \frac{\sqrt{ax+b}}{x^m}\,dx = \frac{-(ax+b)^{3/2}}{(m-1)bx^{m-1}} - \frac{(2m-5)a}{(2m-2)b}\int \frac{\sqrt{ax+b}}{x^{m-1}}\,dx$$

1.2.16 $$\int (ax+b)^{m/2}\,dx = \frac{2(ax+b)^{(m+2)/2}}{a(m+2)}$$

1.2.17 $$\int x(ax+b)^{m/2}\,dx = \frac{2(ax+b)^{(m+4)/2}}{a^2(m+4)} - \frac{2b(ax+b)^{(m+2)/2}}{a^2(m+2)}$$

1.2.18 $$\int x^2(ax+b)^{m/2}\,dx = \frac{2(ax+b)^{(m+6)/2}}{a^3(m+6)} - \frac{4b(ax+b)^{(m+4)/2}}{a^3(m+4)} + \frac{2b^2(ax+b)^{(m+2)/2}}{a^3(m+2)}$$

1.2.19 $$\int \frac{(ax+b)^{m/2}}{x}\,dx = \frac{2(ax+b)^{m/2}}{m} + b\int \frac{(ax+b)^{(m-2)/2}}{x}\,dx$$

1.2.20 $$\int \frac{(ax+b)^{m/2}}{x^2}\,dx = -\frac{(ax+b)^{(m+2)/2}}{bx} + \frac{ma}{2b}\int \frac{(ax+b)^{m/2}}{x}\,dx$$

1.2.21 $$\int \frac{dx}{x(ax+b)^{m/2}} = \frac{2}{(m-2)b(ax+b)^{(m-2)/2}} + \frac{1}{b}\int \frac{dx}{x(ax+b)^{(m-2)/2}}$$

(3) INTEGRALES QUE CONTIENEN $ax+b$ Y $px+q$

1.3.1 $$\int \frac{dx}{(ax+b)(px+q)} = \frac{1}{bp-aq}\ln\left(\frac{px+q}{ax+b}\right)$$

1.3.2 $$\int \frac{x\,dx}{(ax+b)(px+q)} = \frac{1}{bp-aq}\left\{\frac{b}{a}\ln(ax+b) - \frac{q}{p}\ln(px+q)\right\}$$

1.3.3 $$\int \frac{dx}{(ax+b)^2(px+q)} = \frac{1}{bp-aq}\left\{\frac{1}{ax+b} + \frac{p}{bp-aq}\ln\left(\frac{px+q}{ax+b}\right)\right\}$$

1.3.4 $$\int \frac{x\,dx}{(ax+b)^2(px+q)} = \frac{1}{bp-aq}\left\{\frac{q}{bp-aq}\ln\left(\frac{ax+b}{px+q}\right) - \frac{b}{a(ax+b)}\right\}$$

1.3.5 $$\int \frac{x^2\,dx}{(ax+b)^2(px+q)} = \frac{b^2}{(bp-aq)a^2(ax+b)} + \frac{1}{(bp-aq)^2}\left\{\frac{q^2}{p}\ln(px+q) + \frac{b(bp-2aq)}{a^2}\ln(ax+b)\right\}$$

1.3.6 $$\int \frac{dx}{(ax+b)^m(px+q)^n} = \frac{-1}{(n-1)(bp-aq)}\left\{\frac{1}{(ax+b)^{m-1}(px+q)^{n-1}} + a(m+n-2)\int \frac{dx}{(ax+b)^m(px+q)^{n-1}}\right\}$$

1.3.7 $$\int \frac{ax+b}{px+q}\,dx = \frac{ax}{p} + \frac{bp-aq}{p^2}\ln(px+q)$$

1.3.8 $$\int \frac{(ax+b)^m}{(px+q)^n}\,dx = \begin{cases} \dfrac{-1}{(n-1)(bp-aq)}\left\{\dfrac{(ax+b)^{m+1}}{(px+q)^{n-1}} + (n-m-2)a\displaystyle\int \dfrac{(ax+b)^m}{(px+q)^{n-1}}\,dx\right\} \\ \dfrac{-1}{(n-m-1)p}\left\{\dfrac{(ax+b)^m}{(px+q)^{n-1}} + m(bp-aq)\displaystyle\int \dfrac{(ax+b)^{m-1}}{(px+q)^n}\,dx\right\} \\ \dfrac{-1}{(n-1)p}\left\{\dfrac{(ax+b)^m}{(px+q)^{n-1}} - ma\displaystyle\int \dfrac{(ax+b)^{m-1}}{(px+q)^{n-1}}\,dx\right\} \end{cases}$$

(4) INTEGRALES QUE CONTIENEN $\sqrt{ax+b}$ Y $px+q$

1.4.1 $$\int \frac{px+q}{\sqrt{ax+b}}\,dx = \frac{2(apx+3aq-2bp)}{3a^2}\sqrt{ax+b}$$

1.4.2 $$\int \frac{dx}{(px+q)\sqrt{ax+b}} = \begin{cases} \dfrac{1}{\sqrt{bp-aq}\sqrt{p}} \ln\left(\dfrac{\sqrt{p(ax+b)}-\sqrt{bp-aq}}{\sqrt{p(ax+b)}+\sqrt{bp-aq}}\right) \\ \dfrac{2}{\sqrt{aq-bp}\sqrt{p}} \operatorname{tg}^{-1}\sqrt{\dfrac{p(ax+b)}{aq-bp}} \end{cases}$$

1.4.3 $$\int \frac{\sqrt{ax+b}}{px+q}\,dx = \begin{cases} \dfrac{2\sqrt{ax+b}}{p} + \dfrac{\sqrt{bp-aq}}{p\sqrt{p}} \ln\left(\dfrac{\sqrt{p(ax+b)}-\sqrt{bp-aq}}{\sqrt{p(ax+b)}+\sqrt{bp-aq}}\right) \\ \dfrac{2\sqrt{ax+b}}{2} - \dfrac{2\sqrt{aq-bp}}{p\sqrt{p}} \operatorname{tg}^{-1}\sqrt{\dfrac{p(ax+b)}{aq-bp}} \end{cases}$$

1.4.4 $$\int (px+q)^n\sqrt{ax+b}\,dx = \frac{2(px+q)^{n+1}\sqrt{ax+b}}{(2n+3)p} + \frac{bp-aq}{(2n+3)p}\int \frac{(px+q)^n}{\sqrt{ax+b}}\,dx$$

1.4.5 $$\int \frac{dx}{(px+q)^n\sqrt{ax+b}} = \frac{\sqrt{ax+b}}{(n-1)(aq-bp)(px+q)^{n-1}} + \frac{(2n-3)a}{2(n-1)(aq-bp)}\int \frac{dx}{(px+q)^{n-1}\sqrt{ax+b}}$$

1.4.6 $$\int \frac{(px+q)^n}{\sqrt{ax+b}}\,dx = \frac{2(px+q)^n\sqrt{ax+b}}{(2n+1)a} + \frac{2n(aq-bp)}{(2n+1)a}\int \frac{(px+q)^{n-1}\,dx}{\sqrt{ax+b}}$$

1.4.7 $$\int \frac{\sqrt{ax+b}}{(px+q)^n}\,dx = \frac{-\sqrt{ax+b}}{(n-1)p(px+q)^{n-1}} + \frac{a}{2(n-1)p}\int \frac{dx}{(px+q)^{n-1}\sqrt{ax+b}}$$

(5) INTEGRALES QUE CONTIENEN $\sqrt{ax+b}$ Y $\sqrt{px+q}$

1.5.1 $$\int \frac{dx}{\sqrt{(ax+b)(px+q)}} = \begin{cases} \dfrac{2}{\sqrt{ap}} \ln\left(\sqrt{a(px+q)}+\sqrt{p(ax+b)}\right) \\ \dfrac{2}{\sqrt{-ap}} \operatorname{tg}^{-1}\sqrt{\dfrac{-p(ax+b)}{a(px+q)}} \end{cases}$$

1.5.2 $$\int \frac{x\,dx}{\sqrt{(ax+b)(px+q)}} = \frac{\sqrt{(ax+b)(px+q)}}{ap} - \frac{bp+aq}{2ap}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

1.5.3 $$\int \sqrt{(ax+b)(px+q)}\,dx = \frac{2apx+bp+aq}{4ap}\sqrt{(ax+b)(px+q)} - \frac{(bp-aq)^2}{8ap}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

1.5.4 $$\int \sqrt{\frac{px+q}{ax+b}}\,dx = \frac{\sqrt{(ax+b)(px+q)}}{a} + \frac{aq-bp}{2a}\int \frac{dx}{\sqrt{(ax+b)(px+q)}}$$

1.5.5 $$\int \frac{dx}{(px+q)\sqrt{(ax+b)(px+q)}} = \frac{2\sqrt{ax+b}}{(aq-bp)\sqrt{px+q}}$$

(6) INTEGRALES QUE CONTIENEN $x^2 + a^2$

1.6.1 $$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\,\mathrm{tg}^{-1}\frac{x}{a}$$

1.6.2 $$\int \frac{x\,dx}{x^2+a^2} = \frac{1}{2}\ln(x^2+a^2)$$

1.6.3 $$\int \frac{x^2\,dx}{x^2+a^2} = x - a\,\mathrm{tg}^{-1}\frac{x}{a}$$

1.6.4 $$\int \frac{x^3\,dx}{x^2+a^2} = \frac{x^2}{2} - \frac{a^2}{2}\ln(x^2+a^2)$$

1.6.5 $$\int \frac{dx}{x(x^2+a^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{x^2+a^2}\right)$$

1.6.6 $$\int \frac{dx}{x^2(x^2+a^2)} = -\frac{1}{a^2x} - \frac{1}{a^3}\,\mathrm{tg}^{-1}\frac{x}{a}$$

1.6.7 $$\int \frac{dx}{x^3(x^2+a^2)} = -\frac{1}{2a^2x^2} - \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2+a^2}\right)$$

1.6.8 $$\int \frac{dx}{(x^2+a^2)^2} = \frac{x}{2a^2(x^2+a^2)} + \frac{1}{2a^3}\,\mathrm{tg}^{-1}\frac{x}{a}$$

1.6.9 $$\int \frac{x\,dx}{(x^2+a^2)^2} = \frac{-1}{2(x^2+a^2)}$$

1.6.10 $$\int \frac{x^2\,dx}{(x^2+a^2)^2} = \frac{-x}{2(x^2+a^2)} + \frac{1}{2a}\,\mathrm{tg}^{-1}\frac{x}{a}$$

1.6.11 $$\int \frac{x^3\,dx}{(x^2+a^2)^2} = \frac{a^2}{2(x^2+a^2)} + \frac{1}{2}\ln(x^2+a^2)$$

1.6.12 $$\int \frac{dx}{x(x^2+a^2)^2} = \frac{1}{2a^2(x^2+a^2)} + \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2+a^2}\right)$$

1.6.13 $$\int \frac{dx}{x^2(x^2+a^2)^2} = -\frac{1}{a^4x} - \frac{x}{2a^4(x^2+a^2)} - \frac{3}{2a^5}\,\mathrm{tg}^{-1}\frac{x}{a}$$

1.6.14 $\displaystyle\int \frac{dx}{x^3(x^2+a^2)^2} = -\frac{1}{2a^4x^2} - \frac{1}{2a^4(x^2+a^2)} - \frac{1}{a^6}\ln\left(\frac{x^2}{x^2+a^2}\right)$

1.6.15 $\displaystyle\int \frac{dx}{(x^2+a^2)^n} = \frac{x}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2}\int \frac{dx}{(x^2+a^2)^{n-1}}$

1.6.16 $\displaystyle\int \frac{x\,dx}{(x^2+a^2)^n} = \frac{-1}{2(n-1)(x^2+a^2)^{n-1}}$

1.6.17 $\displaystyle\int \frac{dx}{x(x^2+a^2)^n} = \frac{1}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{1}{a^2}\int \frac{dx}{x(x^2+a^2)^{n-1}}$

1.6.18 $\displaystyle\int \frac{x^m\,dx}{(x^2+a^2)^n} = \int \frac{x^{m-2}\,dx}{(x^2+a^2)^{n-1}} - a^2\int \frac{x^{m-2}\,dx}{(x^2+a^2)^n}$

1.6.19 $\displaystyle\int \frac{dx}{x^m(x^2+a^2)^n} = \frac{1}{a^2}\int \frac{dx}{x^m(x^2+a^2)^{n-1}} - \frac{1}{a^2}\int \frac{dx}{x^{m-2}(x^2+a^2)^n}$

(7) INTEGRALES QUE CONTIENEN $x^2 - a^2$, $x^2 > a^2$

1.7.1 $\displaystyle\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)$ o $\displaystyle -\frac{1}{a}\coth^{-1}\frac{x}{a}$

1.7.2 $\displaystyle\int \frac{x\,dx}{x^2-a^2} = \frac{1}{2}\ln(x^2-a^2)$

1.7.3 $\displaystyle\int \frac{x^2\,dx}{x^2-a^2} = x + \frac{a}{2}\ln\left(\frac{x-a}{x+a}\right)$

1.7.4 $\displaystyle\int \frac{x^3\,dx}{x^2-a^2} = \frac{x^2}{2} + \frac{a^2}{2}\ln(x^2-a^2)$

1.7.5 $\displaystyle\int \frac{dx}{x(x^2-a^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2-a^2}{x^2}\right)$

1.7.6 $\displaystyle\int \frac{dx}{x^2(x^2-a^2)} = \frac{1}{a^2x} + \frac{1}{2a^3}\ln\left(\frac{x-a}{x+a}\right)$

1.7.7 $\displaystyle\int \frac{dx}{x^3(x^2-a^2)} = \frac{1}{2a^2x^2} - \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2-a^2}\right)$

1.7.8 $\displaystyle\int \frac{dx}{(x^2-a^2)^2} = \frac{-x}{2a^2(x^2-a^2)} - \frac{1}{4a^3}\ln\left(\frac{x-a}{x+a}\right)$

1.7.9 $\displaystyle\int \frac{x\,dx}{(x^2-a^2)^2} = \frac{-1}{2(x^2-a^2)}$

1.7.10 $\displaystyle\int \frac{x^2\,dx}{(x^2-a^2)^2} = \frac{-x}{2(x^2-a^2)} + \frac{1}{4a}\ln\left(\frac{x-a}{x+a}\right)$

1.7.11 $\displaystyle\int \frac{x^3\,dx}{(x^2-a^2)^2} = \frac{-a}{2(x^2-a^2)} + \frac{1}{2}\ln(x^2-a^2)$

1.7.12 $\displaystyle\int \frac{dx}{x(x^2-a^2)^2} = \frac{-1}{2a^2(x^2-a^2)} + \frac{1}{2a^4}\ln\left(\frac{x^2}{x^2-a^2}\right)$

1.7.13 $\displaystyle\int \frac{dx}{x^2(x^2-a^2)^2} = -\frac{1}{a^4x} - \frac{x}{2a^4(x^2-a^2)} - \frac{3}{4a^5}\ln\left(\frac{x-a}{x+a}\right)$

1.7.14 $\displaystyle\int \frac{dx}{x^3(x^2-a^2)^2} = -\frac{1}{2a^4x^2} - \frac{1}{2a^4(x^2-a^2)} + \frac{1}{a^6}\ln\left(\frac{x^2}{x^2-a^2}\right)$

1.7.15 $\displaystyle\int \frac{dx}{(x^2-a^2)^n} = \frac{-x}{2(n-1)a^2(x^2-a^2)^{n-1}} - \frac{2n-3}{(2n-2)a^2}\int \frac{dx}{(x^2-a^2)^{n-1}}$

1.7.16 $\displaystyle\int \frac{x\,dx}{(x^2-a^2)^n} = \frac{-1}{2(n-1)(x^2-a^2)^{n-1}}$

1.7.17 $\displaystyle\int \frac{dx}{x(x^2-a^2)^n} = \frac{-1}{2(n-1)a^2(x^2-a^2)^{n-1}} - \frac{1}{a^2}\int \frac{dx}{x(x^2-a^2)^{n-1}}$

1.7.18 $\displaystyle\int \frac{x^m\,dx}{(x^2-a^2)^n} = \int \frac{x^{m-2}\,dx}{(x^2-a^2)^{n-1}} + a^2\int \frac{x^{m-2}\,dx}{(x^2-a^2)^n}$

1.7.19 $\displaystyle\int \frac{dx}{x^m(x^2-a^2)^n} = \frac{1}{a^2}\int \frac{dx}{x^{m-2}(x^2-a^2)^n} - \frac{1}{a^2}\int \frac{dx}{x^m(x^2-a^2)^{n-1}}$

(8) INTEGRALES QUE CONTIENEN $a^2 - x^2$, $x^2 < a^2$

1.8.1 $\displaystyle\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right)$ o $\displaystyle\frac{1}{a}\,\mathrm{th}^{-1}\frac{x}{a}$

1.8.2 $\displaystyle\int \frac{x\,dx}{a^2-x^2} = -\frac{1}{2}\ln(a^2-x^2)$

1.8.3 $\displaystyle\int \frac{x^2\,dx}{a^2-x^2} = -x + \frac{a}{2}\ln\left(\frac{a+x}{a-x}\right)$

1.8.4 $\displaystyle\int \frac{x^3\,dx}{a^2-x^2} = -\frac{x^2}{2} - \frac{a^2}{2}\ln(a^2-x^2)$

1.8.5 $\displaystyle\int \frac{dx}{x(a^2-x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2-x^2}\right)$

1.8.6 $\displaystyle\int \frac{dx}{x^2(a^2-x^2)} = -\frac{1}{a^2x} + \frac{1}{2a^3}\ln\left(\frac{a+x}{a-x}\right)$

1.8.7 $\displaystyle\int \frac{dx}{x^3(a^2-x^2)} = -\frac{1}{2a^2x^2} + \frac{1}{2a^4}\ln\left(\frac{x^2}{a^2-x^2}\right)$

1.8.8 $$\int \frac{dx}{(a^2 - x^2)^2} = \frac{x}{2a^2(a^2 - x^2)} + \frac{1}{4a^3} \ln\left(\frac{a + x}{a - x}\right)$$

1.8.9 $$\int \frac{x\,dx}{(a^2 - x^2)^2} = \frac{1}{2(a^2 - x^2)}$$

1.8.10 $$\int \frac{x^2\,dx}{(a^2 - x^2)^2} = \frac{x}{2(a^2 - x^2)} - \frac{1}{4a} \ln\left(\frac{a + x}{a - x}\right)$$

1.8.11 $$\int \frac{x^3\,dx}{(a^2 - x^2)^2} = \frac{a^2}{2(a^2 - x^2)} + \frac{1}{2} \ln(a^2 - x^2)$$

1.8.12 $$\int \frac{dx}{x(a^2 - x^2)^2} = \frac{1}{2a^2(a^2 - x^2)} + \frac{1}{2a^4} \ln\left(\frac{x^2}{a^2 - x^2}\right)$$

1.8.13 $$\int \frac{dx}{x^2(a^2 - x^2)^2} = \frac{-1}{a^4 x} + \frac{x}{2a^4(a^2 - x^2)} + \frac{3}{4a^5} \ln\left(\frac{a + x}{a - x}\right)$$

1.8.14 $$\int \frac{dx}{x^3(a^2 - x^2)^2} = \frac{-1}{2a^4 x^2} + \frac{1}{2a^4(a^2 - x^2)} + \frac{1}{a^6} \ln\left(\frac{x^2}{a^2 - x^2}\right)$$

1.8.15 $$\int \frac{dx}{(a^2 - x^2)^n} = \frac{x}{2(n-1)a^2(a^2 - x^2)^{n-1}} + \frac{2n - 3}{(2n - 2)a^2} \int \frac{dx}{(a^2 - x^2)^{n-1}}$$

1.8.16 $$\int \frac{x\,dx}{(a^2 - x^2)^n} = \frac{1}{2(n-1)(a^2 - x^2)^{n-1}}$$

1.8.17 $$\int \frac{dx}{x(a^2 - x^2)^n} = \frac{1}{2(n-1)a^2(a^2 - x^2)^{n-1}} + \frac{1}{a^2} \int \frac{dx}{x^2(a^2 - x^2)^{n-1}}$$

1.8.18 $$\int \frac{x^m\,dx}{(a^2 - x^2)^n} = a^2 \int \frac{x^{m-2}\,dx}{(a^2 - x^2)^n} - \int \frac{x^{m-2}\,dx}{(a^2 - x^2)^{n-1}}$$

1.8.19 $$\int \frac{dx}{x^m(a^2 - x^2)^n} = \frac{1}{a^2} \int \frac{dx}{x^m(a^2 - x^2)^{n-1}} + \frac{1}{a^2} \int \frac{dx}{x^{m-2}(a^2 - x^2)^n}$$

(9) INTEGRALES QUE CONTIENEN $\sqrt{x^2 + a^2}$

1.9.1 $$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2}) \quad \text{o} \quad \operatorname{sh}^{-1} \frac{x}{a}$$

1.9.2 $$\int \frac{x\,dx}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2}$$

1.9.3 $$\int \frac{x^2\,dx}{\sqrt{x^2 + a^2}} = \frac{x\sqrt{x^2 + a^2}}{2} - \frac{a^2}{2} \ln(x + \sqrt{x^2 + a^2})$$

1.9.4 $$\int \frac{x^3\,dx}{\sqrt{x^2+a^2}} = \frac{(x^2+a^2)^{3/2}}{3} - a^2\sqrt{x^2+a^2}$$

1.9.5 $$\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

1.9.6 $$\int \frac{dx}{x^2\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{a^2x}$$

1.9.7 $$\int \frac{dx}{x^3\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{2a^2x^2} + \frac{1}{2a^3}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

1.9.8 $$\int \sqrt{x^2+a^2}\,dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\ln(x+\sqrt{x^2+a^2})$$

1.9.9 $$\int x\sqrt{x^2+a^2}\,dx = \frac{(x^2+a^2)^{3/2}}{3}$$

1.9.10 $$\int x^2\sqrt{x^2+a^2}\,dx = \frac{x(x^2+a^2)^{3/2}}{4} - \frac{a^2x\sqrt{x^2+a^2}}{8} - \frac{a^4}{8}\ln(x+\sqrt{x^2+a^2})$$

1.9.11 $$\int x^3\sqrt{x^2+a^2}\,dx = \frac{(x^2+a^2)^{5/2}}{5} - \frac{a^2(x^2+a^2)^{3/2}}{3}$$

1.9.12 $$\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2} - a\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

1.9.13 $$\int \frac{\sqrt{x^2+a^2}}{x^2}\,dx = -\frac{\sqrt{x^2+a^2}}{x} + \ln(x+\sqrt{x^2+a^2})$$

1.9.14 $$\int \frac{\sqrt{x^2+a^2}}{x^3}\,dx = -\frac{\sqrt{x^2+a^2}}{2x^2} - \frac{1}{2a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

1.9.15 $$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$

1.9.16 $$\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}}$$

1.9.17 $$\int \frac{x^2\,dx}{(x^2+a^2)^{3/2}} = \frac{-x}{\sqrt{x^2+a^2}} + \ln(x+\sqrt{x^2+a^2})$$

1.9.18 $$\int \frac{x^3\,dx}{(x^2+a^2)^{3/2}} = \sqrt{x^2+a^2} + \frac{a^2}{\sqrt{x^2+a^2}}$$

1.9.19 $$\int \frac{dx}{x(x^2+a^2)^{3/2}} = \frac{1}{a^2\sqrt{x^2+a^2}} - \frac{1}{a^3}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

1.9.20 $$\int \frac{dx}{x^2(x^2+a^2)^{3/2}} = -\frac{\sqrt{x^2+a^2}}{a^4x} - \frac{x}{a^4\sqrt{x^2+a^2}}$$

1.9.21 $$\int \frac{dx}{x^3(x^2+a^2)^{3/2}} = \frac{-1}{2a^2x^2\sqrt{x^2+a^2}} - \frac{3}{2a^4\sqrt{x^2+a^2}} + \frac{3}{2a^5}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

1.9.22 $$\int (x^2+a^2)^{3/2}\,dx = \frac{x(x^2+a^2)^{3/2}}{4} + \frac{3a^2x\sqrt{x^2+a^2}}{8} + \frac{3}{8}a^4\ln(x+\sqrt{x^2+a^2})$$

1.9.23 $$\int x(x^2+a^2)^{3/2}\,dx = \frac{(x^2+a^2)^{5/2}}{5}$$

1.9.24 $$\int x^2(x^2+a^2)^{3/2}\,dx = \frac{x(x^2+a^2)^{5/2}}{6} - \frac{a^2x(x^2+a^2)^{3/2}}{24} - \frac{a^4x\sqrt{x^2+a^2}}{16} - \frac{a^6}{16}\ln(x+\sqrt{x^2+a^2})$$

1.9.25 $$\int x^3(x^2+a^2)^{3/2}\,dx = \frac{(x^2+a^2)^{7/2}}{7} - \frac{a^2(x^2+a^2)^{5/2}}{5}$$

1.9.26 $$\int \frac{(x^2+a^2)^{3/2}}{x}\,dx = \frac{(x^2+a^2)^{3/2}}{3} + a^2\sqrt{x^2+a^2} - a^3\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

1.9.27 $$\int \frac{(x^2+a^2)^{3/2}}{x^2}\,dx = -\frac{(x^2+a^2)^{3/2}}{x} + \frac{3x\sqrt{x^2+a^2}}{2} + \frac{3}{2}a^2\ln(x+\sqrt{x^2+a^2})$$

1.9.28 $$\int \frac{(x^2+a^2)^{3/2}}{x^3}\,dx = -\frac{(x^2+a^2)^{3/2}}{2x^2} + \frac{3}{2}\sqrt{x^2+a^2} - \frac{3}{2}a\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

(10) INTEGRALES QUE CONTIENEN $\sqrt{x^2-a^2}$

1.10.1 $$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln(x+\sqrt{x^2-a^2}), \qquad \int \frac{x\,dx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2}$$

1.10.2 $$\int \frac{x^2\,dx}{\sqrt{x^2-a^2}} = \frac{x\sqrt{x^2-a^2}}{2} + \frac{a^2}{2}\ln(x+\sqrt{x^2-a^2})$$

1.10.3 $$\int \frac{x^3\,dx}{\sqrt{x^2-a^2}} = \frac{(x^2-a^2)^{3/2}}{3} + a^2\sqrt{x^2-a^2}$$

1.10.4 $$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|$$

1.10.5 $$\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2x}$$

1.10.6 $$\int \frac{dx}{x^3\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{2a^2x^2} + \frac{1}{2a^3}\sec^{-1}\left|\frac{x}{a}\right|$$

1.10.7 $$\int \sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\ln(x+\sqrt{x^2-a^2})$$

1.10.8 $$\int x\sqrt{x^2-a^2}\,dx = \frac{(x^2-a^2)^{3/2}}{3}$$

1.10.9 $$\int x^2\sqrt{x^2-a^2}\,dx = \frac{x(x^2-a^2)^{3/2}}{4} + \frac{a^2x\sqrt{x^2-a^2}}{8} - \frac{a^4}{8}\ln(x+\sqrt{x^2-a^2})$$

1.10.10 $$\int x^3\sqrt{x^2-a^2}\,dx = \frac{(x^2-a^2)^{5/2}}{5} + \frac{a^2(x^2-a^2)^{3/2}}{3}$$

1.10.11 $$\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2} - a\sec^{-1}\left|\frac{x}{a}\right|$$

1.10.12 $$\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln(x+\sqrt{x^2-a^2})$$

1.10.13 $$\int \frac{\sqrt{x^2-a^2}}{x^3}\,dx = -\frac{\sqrt{x^2-a^2}}{2x^2} + \frac{1}{2a}\sec^{-1}\left|\frac{x}{a}\right|$$

1.10.14 $$\int \frac{dx}{(x^2-a^2)^{3/2}} = -\frac{x}{a^2\sqrt{x^2-a^2}}$$

1.10.15 $$\int \frac{x\,dx}{(x^2-a^2)^{3/2}} = \frac{-1}{\sqrt{x^2-a^2}}$$

1.10.16 $$\int \frac{x^2\,dx}{(x^2-a^2)^{3/2}} = -\frac{x}{\sqrt{x^2-a^2}} + \ln(x+\sqrt{x^2-a^2})$$

1.10.17 $$\int \frac{x^3\,dx}{(x^2-a^2)^{3/2}} = \sqrt{x^2-a^2} - \frac{a^2}{\sqrt{x^2-a^2}}$$

1.10.18 $$\int \frac{dx}{x(x^2-a^2)^{3/2}} = \frac{-1}{a^2\sqrt{x^2-a^2}} - \frac{1}{a^3}\sec^{-1}\left|\frac{x}{a}\right|$$

1.10.19 $$\int \frac{dx}{x^2(x^2-a^2)^{3/2}} = -\frac{\sqrt{x^2-a^2}}{a^4x} - \frac{x}{a^4\sqrt{x^2-a^2}}$$

1.10.20 $$\int \frac{dx}{x^3(x^2-a^2)^{3/2}} = \frac{1}{2a^2x^2\sqrt{x^2-a^2}} - \frac{3}{2a^4\sqrt{x^2-a^2}} - \frac{3}{2a^5}\sec^{-1}\left|\frac{x}{a}\right|$$

1.10.21 $$\int (x^2-a^2)^{3/2}\,dx = \frac{x(x^2-a^2)^{3/2}}{4} - \frac{3a^2x\sqrt{x^2-a^2}}{8} + \frac{3}{8}a^4\ln(x+\sqrt{x^2+a^2})$$

1.10.22 $\displaystyle\int x(x^2-a^2)^{3/2}\,dx = \frac{(x^2-a^2)^{5/2}}{5}$

1.10.23 $\displaystyle\int x^2(x^2-a^2)^{3/2}\,dx = \frac{x(x^2-a^2)^{5/2}}{6} + \frac{a^2x(x^2-a^2)^{3/2}}{24} - \frac{a^4x\sqrt{x^2-a^2}}{16} + \frac{a^6}{16}\ln(x+\sqrt{x^2-a^2})$

1.10.24 $\displaystyle\int x^3(x^2-a^2)^{3/2}\,dx = \frac{(x^2-a^2)^{7/2}}{7} + \frac{a^2(x^2-a^2)^{5/2}}{5}$

1.10.25 $\displaystyle\int \frac{(x^2-a^2)^{3/2}}{x}\,dx = \frac{(x^2-a^2)^{3/2}}{3} - a^2\sqrt{x^2-a^2} + a^3\sec^{-1}\left|\frac{x}{a}\right|$

1.10.26 $\displaystyle\int \frac{(x^2-a^2)^{3/2}}{x^2}\,dx = -\frac{(x^2-a^2)^{3/2}}{x} + \frac{3x\sqrt{x^2-a^2}}{2} - \frac{3}{2}a^2\ln(x+\sqrt{x^2-a^2})$

1.10.27 $\displaystyle\int \frac{(x^2-a^2)^{3/2}}{x^3}\,dx = -\frac{(x^2-a^2)^{3/2}}{2x^2} + \frac{3\sqrt{x^2-a^2}}{2} - \frac{3}{2}a\sec^{-1}\left|\frac{x}{a}\right|$

(11) INTEGRALES QUE CONTIENEN $\sqrt{a^2-x^2}$

1.11.1 $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{sen}^{-1}\frac{x}{a}$

1.11.2 $\displaystyle\int \frac{x\,dx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2}$

1.11.3 $\displaystyle\int \frac{x^2\,dx}{\sqrt{a^2-x^2}} = -\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{x}{a}$

1.11.4 $\displaystyle\int \frac{x^3\,dx}{\sqrt{a^2-x^2}} = \frac{(a^2-x^3)^{3/2}}{3} - a^2\sqrt{a^2-x^2}$

1.11.5 $\displaystyle\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

1.11.6 $\displaystyle\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x}$

1.11.7 $\displaystyle\int \frac{dx}{x^3\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{2a^2x^2} - \frac{1}{2a^3}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

1.11.8 $\displaystyle\int \sqrt{a^2-x^2}\,dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{x}{a}$

1.11.9 $\displaystyle\int x\sqrt{a^2-x^2}\,dx = -\frac{(a^2-x^2)^{3/2}}{3}$

1.11.10 $$\int x^2\sqrt{a^2-x^2}\,dx = -\frac{x(a^2-x^2)^{3/2}}{4}+\frac{a^2x\sqrt{a^2-x^2}}{8}+\frac{a^4}{8}\operatorname{sen}^{-1}\frac{x}{a}$$

1.11.11 $$\int x^3\sqrt{a^2-x^2}\,dx = \frac{(a^2-x^2)^{5/2}}{5}-\frac{a^2(a^2-x^2)^{3/2}}{3}$$

1.11.12 $$\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2}-a\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

1.11.13 $$\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\frac{\sqrt{a^2-x^2}}{x}-\operatorname{sen}^{-1}\frac{x}{a}$$

1.11.14 $$\int \frac{\sqrt{a^2-x^2}}{x^3}\,dx = -\frac{\sqrt{a^2-x^2}}{2x^2}+\frac{1}{2a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

1.11.15 $$\int \frac{dx}{(a^2-x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2-x^2}}$$

1.11.16 $$\int \frac{x\,dx}{(a^2-x^2)^{3/2}} = \frac{1}{\sqrt{a^2-x^2}}$$

1.11.17 $$\int \frac{x^2\,dx}{(a^2-x^2)^{3/2}} = \frac{x}{\sqrt{a^2-x^2}}-\operatorname{sen}^{-1}\frac{x}{a}$$

1.11.18 $$\int \frac{x^3\,dx}{(a^2-x^2)^{3/2}} = \sqrt{a^2-x^2}+\frac{a^2}{\sqrt{a^2-x^2}}$$

1.11.19 $$\int \frac{dx}{x(a^2-x^2)^{3/2}} = \frac{1}{a^2\sqrt{a^2-x^2}}-\frac{1}{a^3}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

1.11.20 $$\int \frac{dx}{x^2(a^2-x^2)^{3/2}} = -\frac{\sqrt{a^2-x^2}}{a^4x}+\frac{x}{a^4\sqrt{a^2-x^2}}$$

1.11.21 $$\int \frac{dx}{x^3(a^2-x^2)^{3/2}} = \frac{-1}{2a^2x^2\sqrt{a^2-x^2}}+\frac{3}{2a^4\sqrt{a^2-x^2}}-\frac{3}{2a^5}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

1.11.22 $$\int (a^2-x^2)^{3/2}\,dx = \frac{x(a^2-x^2)^{3/2}}{4}+\frac{3a^2x\sqrt{a^2-x^2}}{8}+\frac{3}{8}a^4\operatorname{sen}^{-1}\frac{x}{a}$$

1.11.23 $$\int x(a^2-x^2)^{3/2}\,dx = -\frac{(a^2-x^2)^{5/2}}{5}$$

1.11.24 $$\int x^2(a^2-x^2)^{3/2}\,dx = -\frac{x(a^2-x^2)^{5/2}}{6}+\frac{a^2x(a^2-x^2)^{3/2}}{24}+\frac{a^4x\sqrt{a^2-x^2}}{16}+\frac{a^6}{16}\operatorname{sen}^{-1}\frac{x}{a}$$

1.11.25 $$\int x^3(a^2-x^2)^{3/2}\,dx = \frac{(a^2-x^2)^{7/2}}{7}-\frac{a^2(a^2-x^2)^{5/2}}{5}$$

1.11.26 $\displaystyle\int \frac{(a^2 - x^2)^{3/2}}{x}\,dx = \frac{(a^2 - x^2)^{3/2}}{3} + a^2\sqrt{a^2 - x^2} - a^3 \ln\left(\frac{a + \sqrt{a^2 - x^2}}{x}\right)$

1.11.27 $\displaystyle\int \frac{(a^2 - x^2)^{3/2}}{x^2}\,dx = -\frac{(a^2 - x^2)^{3/2}}{x} - \frac{3x\sqrt{a^2 - x^2}}{2} + \frac{3}{2}a^2 \operatorname{sen}^{-1}\frac{x}{a}$

1.11.28 $\displaystyle\int \frac{(a^2 - x^2)^{3/2}}{x^3}\,dx = -\frac{(a^2 - x^2)^{3/2}}{2x^2} - \frac{3\sqrt{a^2 - x^2}}{2} + \frac{3}{2}a \ln\left(\frac{a + \sqrt{a^2 - x^2}}{x}\right)$

(12) INTEGRALES QUE CONTIENEN $ax^2 + bx + c$

1.12.1 $\displaystyle\int \frac{dx}{ax^2 + bx + c} = \begin{cases} \dfrac{2}{\sqrt{4ac - b^2}}\operatorname{tg}^{-1}\dfrac{2ax + b}{\sqrt{4ac - b^2}} \\[2ex] \dfrac{1}{\sqrt{b^2 - 4ac}}\ln\left(\dfrac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}}\right) \end{cases}$

Si $b^2 = 4ac$, entonces $ax^2 + bx + c = a(x + b/2a)^2$ y se pueden utilizar los resultados de las integrales 1.1.6 a 1.1.10 y 1.1.14 a 1.1.17. Si $b = 0$, utilícense los resultados de las páginas 253-254. Si $a = 0$ o si $c = 0$, pueden usarse los resultados de las páginas 249-250.

1.12.2 $\displaystyle\int \frac{x\,dx}{ax^2 + bx + c} = \frac{1}{2a}\ln(ax^2 + bx + c) - \frac{b}{2a}\int \frac{dx}{ax^2 + bx + c}$

1.12.3 $\displaystyle\int \frac{x^2\,dx}{ax^2 + bx + c} = \frac{x}{a} - \frac{b}{2a^2}\ln(ax^2 + bx + c) + \frac{b^2 - 2ac}{2a^2}\int \frac{dx}{ax^2 + bx + c}$

1.12.4 $\displaystyle\int \frac{x^m\,dx}{ax^2 + bx + c} = \frac{x^{m-1}}{(m-1)a} - \frac{c}{a}\int \frac{x^{m-2}\,dx}{ax^2 + bx + c} - \frac{b}{a}\int \frac{x^{m-1}\,dx}{ax^2 + bx + c}$

1.12.5 $\displaystyle\int \frac{dx}{x(ax^2 + bx + c)} = \frac{1}{2c}\ln\left(\frac{x^2}{ax^2 + bx + c}\right) - \frac{b}{2c}\int \frac{dx}{ax^2 + bx + c}$

1.12.6 $\displaystyle\int \frac{dx}{x^2(ax^2 + bx + c)} = \frac{b}{2c^2}\ln\left(\frac{ax^2 + bx + c}{x^2}\right) - \frac{1}{cx} + \frac{b^2 - 2ac}{2c^2}\int \frac{dx}{ax^2 + bx + c}$

1.12.7 $\displaystyle\int \frac{dx}{x^n(ax^2 + bx + c)} = -\frac{1}{(n-1)cx^{n-1}} - \frac{b}{c}\int \frac{dx}{x^{n-1}(ax^2 + bx + c} - \frac{a}{c}\int \frac{dx}{x^{n-2}(ax^2 + bx + c)}$

1.12.8 $\displaystyle\int \frac{dx}{(ax^2 + bx + c)^2} = \frac{2ax + b}{(4ac - b^2)(ax^2 + bx + c)} + \frac{2a}{4ac - b^2}\int \frac{dx}{ax^2 + bx + c}$

1.12.9 $\displaystyle\int \frac{x\,dx}{(ax^2 + bx + c)^2} = -\frac{bx + 2c}{(4ac - b^2)(ax^2 + bx + c)} - \frac{b}{4ac - b^2}\int \frac{dx}{ax^2 + bx + c}$

1.12.10 $\displaystyle\int \frac{x^2\,dx}{(ax^2 + bx + c)^2} = \frac{(b^2 - 2ac)x + bc}{a(4ac + b^2)(ax^2 + bx + c)} + \frac{2c}{4ac - b^2}\int \frac{dx}{ax^2 + bx + c}$

1.12.11 $$\int \frac{x^m\,dx}{(ax^2+bx+c)^n} = -\frac{x^{m-1}}{(2n-m-1)a(ax^2+bx+c)^{n-1}} + \frac{(m-1)c}{(2n-m-1)a}\int \frac{x^{m-2}\,dx}{(ax^2+bx+c)^n} - \frac{(n-m)b}{(2n-m-1)a}\int \frac{x^{m-1}\,dx}{(ax^2+bx+c)^n}$$

1.12.12 $$\int \frac{x^{2n-1}\,dx}{(ax^2+bx+c)^n} = \frac{1}{a}\int \frac{x^{2n-3}\,dx}{(ax^2+bx+c)^{n-1}} - \frac{c}{a}\int \frac{x^{2n-3}\,dx}{(ax^2+bx+c)^n} - \frac{b}{a}\int \frac{x^{2n-2}\,dx}{(ax^2+bx+c)^n}$$

1.12.13 $$\int \frac{dx}{x(ax^2+bx+c)^2} = \frac{1}{2c(ax^2+bx+c)} - \frac{b}{2c}\int \frac{dx}{(ax^2+bx+c)^2} + \frac{1}{c}\int \frac{dx}{x(ax^2+bx+c)}$$

1.12.14 $$\int \frac{dx}{x^2(ax^2+bx+c)^2} = -\frac{1}{cx(ax^2+bx+c)} - \frac{3a}{c}\int \frac{dx}{(ax^2+bx+c)^2} - \frac{2b}{c}\int \frac{dx}{x(ax^2+bx+c)^2}$$

1.12.15 $$\int \frac{dx}{x^m(ax^2+bx+c)^n} = -\frac{1}{(m-1)cx^{m-1}(ax^2+bx+c)^{n-1}} - \frac{(m+2n-3)a}{(m-1)c}\int \frac{dx}{x^{m-2}(ax^2+bx+c)^n} - \frac{(m+n-2)b}{(m-1)c}\int \frac{dx}{x^{m-1}(ax^2+bx+c)^n}$$

(13) INTEGRALES QUE CONTIENEN $\sqrt{ax^2+bx+c}$

En los resultados de este apartado, si $b^2 = 4ac$, es $\sqrt{ax^2+bx+c} = \sqrt{a}(x + b/2a)$, y se pueden utilizar los resultados del Apartado 1 de esta tabla de integrales. Si $b = 0$, pueden aplicarse los resultados del Apartado 9 de esta tabla. Si $a = 0$ o $c = 0$, pueden usarse las integrales de los Apartados 2 y 5 de esta tabla.

1.13.1 $$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \begin{cases} \dfrac{1}{\sqrt{a}} \ln\left(2\sqrt{a}\sqrt{ax^2+bx+c} + 2ax + b\right) \\ -\dfrac{1}{\sqrt{-a}} \operatorname{sen}^{-1}\left(\dfrac{2ax+b}{\sqrt{b^2-4ac}}\right) \quad \text{o} \quad \dfrac{1}{\sqrt{a}} \operatorname{sh}^{-1}\left(\dfrac{2ax+b}{\sqrt{4ac-b^2}}\right) \end{cases}$$

1.13.2 $$\int \frac{x\,dx}{\sqrt{ax^2+bx+c}} = \frac{\sqrt{ax^2+bx+c}}{a} - \frac{b}{2a}\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

1.13.3 $$\int \frac{x^2\,dx}{\sqrt{ax^2+bx+c}} = \frac{2ax-3b}{4a^2}\sqrt{ax^2+bx+c} + \frac{3b^2-4ac}{8a^2}\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

1.13.4 $$\int \frac{dx}{x\sqrt{ax^2+bx+c}} = \begin{cases} -\dfrac{1}{\sqrt{c}} \ln\left(\dfrac{2\sqrt{c}\sqrt{ax^2+bx+c} + bx + 2c}{x}\right) \\ \dfrac{1}{\sqrt{-c}} \operatorname{sen}^{-1}\left(\dfrac{bx+2c}{|x|\sqrt{b^2-4ac}}\right) \quad \text{o} \quad -\dfrac{1}{\sqrt{c}} \operatorname{sh}^{-1}\left(\dfrac{bx+2c}{|x|\sqrt{4ac-b^2}}\right) \end{cases}$$

1.13.5 $$\int \frac{dx}{x^2\sqrt{ax^2+bx+c}} = -\frac{\sqrt{ax^2+bx+c}}{cx} - \frac{b}{2c}\int \frac{dx}{x\sqrt{ax^2+bx+c}}$$

1.13.6 $$\int \sqrt{ax^2+bx+c}\,dx = \frac{(2ax+b)\sqrt{ax^2+bx+c}}{4a} + \frac{4ac-b^2}{8a}\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

1.13.7 $$\int x\sqrt{ax^2+bx+c}\,dx = \frac{(ax^2+bx+c)^{3/2}}{3a} - \frac{b(2ax+b)}{8a^2}\sqrt{ax^2+bx+c} - \frac{b(4ac-b^2)}{16a^2}\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

1.13.8 $$\int x^2\sqrt{ax^2+bx+c}\,dx = \frac{6ax-5b}{24a^2}(ax^2+bx+c)^{3/2} + \frac{5b^2-4ac}{16a^2}\int \sqrt{ax^2+bx+c}\,dx$$

1.13.9 $$\int \frac{\sqrt{ax^2+bx+c}}{x}\,dx = \sqrt{ax^2+bx+c} + \frac{b}{2}\int \frac{dx}{\sqrt{ax^2+bx+c}} + c\int \frac{dx}{x\sqrt{ax^2+bx+c}}$$

1.13.10 $$\int \frac{\sqrt{ax^2+bx+c}}{x^2}\,dx = -\frac{\sqrt{ax^2+bx+c}}{x} + a\int \frac{dx}{\sqrt{ax^2+bx+c}} + \frac{b}{2}\int \frac{dx}{x\sqrt{ax^2+bx+c}}$$

1.13.11 $$\int \frac{dx}{(ax^2+bx+c)^{3/2}} = \frac{2(2ax+b)}{(4ac-b^2)\sqrt{ax^2+bx+c}}$$

1.13.12 $$\int \frac{x\,dx}{(ax^2+bx+c)^{3/2}} = \frac{2(bx+2c)}{(b^2-4ac)\sqrt{ax^2+bx+c}}$$

1.13.13 $$\int \frac{x^2\,dx}{(ax^2+bx+c)^{3/2}} = \frac{(2b^2-4ac)x+2bc}{a(4ac-b^2)\sqrt{ax^2+bx+c}} + \frac{1}{a}\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

1.13.14 $$\int \frac{dx}{x(ax^2+bx+c)^{3/2}} = \frac{1}{c\sqrt{ax^2+bx+c}} + \frac{1}{c}\int \frac{dx}{x\sqrt{ax^2+bx+c}} - \frac{b}{2c}\int \frac{dx}{(ax^2+bx+c)^{3/2}}$$

1.13.15 $$\int \frac{dx}{x^2(ax^2+bx+c)^{3/2}} = -\frac{ax^2+2bx+c}{c^2x\sqrt{ax^2+bx+c}} + \frac{b^2-2ac}{2c^2}\int \frac{dx}{(ax^2+bx+c)^{3/2}} - \frac{3b}{2c^2}\int \frac{dx}{x\sqrt{ax^2+bx+c}}$$

1.13.16 $$\int (ax^2+bx+c)^{n+1/2}\,dx = \frac{(2ax+b)(ax^2+bx+c)^{n+1/2}}{4a(n+1)} + \frac{(2n+1)(4ac-b^2)}{8a(n+1)}\int (ax^2+bx+c)^{n-1/2}\,dx$$

1.13.17 $$\int x(ax^2+bx+c)^{n+1/2}\,dx = \frac{(ax^2+bx+c)^{n+3/2}}{a(2n+3)} - \frac{b}{2a}\int (ax^2+bx+c)^{n+1/2}\,dx$$

1.13.18 $$\int \frac{dx}{(ax^2+bx+c)^{n+1/2}} = \frac{2(2ax+b)}{(2n-1)(4ac-b^2)(ax^2+bx+c)^{n-1/2}} + \frac{8a(n-1)}{(2n-1)(4ac-b^2)}\int \frac{dx}{(ax^2+bx+c)^{n-1/2}}$$

1.13.19 $$\int \frac{dx}{x(ax^2+bx+c)^{n+1/2}} = \frac{1}{(2n-1)c(ax^2+bx+c)^{n-1/2}} + \frac{1}{c}\int \frac{dx}{x(ax^2+bx+c)^{n-1/2}} - \frac{b}{2c}\int \frac{dx}{(ax^2+bx+c)^{n+1/2}}$$

(14) INTEGRALES QUE CONTIENEN $x^3 + a^3$

Para fórmulas que contienen $x^3 - a^3$, reemplace a por $-a$.

1.14.1 $$\int \frac{dx}{x^3+a^3} = \frac{1}{6a^2}\ln\frac{(x+a)^2}{x^2-ax+a^2} + \frac{1}{a^2\sqrt{3}}\operatorname{tg}^{-1}\frac{2x-a}{a\sqrt{3}}$$

1.14.2 $$\int \frac{x\,dx}{x^3+a^3} = \frac{1}{6a}\ln\frac{x^2-ax+a^2}{(x+a)^2} + \frac{1}{a\sqrt{3}}\operatorname{tg}^{-1}\frac{2x-a}{a\sqrt{3}}$$

1.14.3 $$\int \frac{x^2\,dx}{x^3+a^3} = \frac{1}{3}\ln(x^3+a^3)$$

1.14.4 $$\int \frac{dx}{x(x^3+a^3)} = \frac{1}{3a^3}\ln\left(\frac{x^3}{x^3+a^3}\right)$$

1.14.5 $$\int \frac{dx}{x^2(x^3+a^3)} = -\frac{1}{a^3x} - \frac{1}{6a^4}\ln\frac{x^2-ax+a^2}{(x+a)^2} - \frac{1}{a^4\sqrt{3}}\operatorname{tg}^{-1}\frac{2x-a}{a\sqrt{3}}$$

1.14.6 $$\int \frac{dx}{(x^3+a^3)^2} = \frac{x}{3a^3(x^3+a^3)} + \frac{1}{9a^5}\ln\frac{(x+a)^2}{x^2-ax+a^2} + \frac{2}{3a^5\sqrt{3}}\operatorname{tg}^{-1}\frac{2x-a}{a\sqrt{3}}$$

1.14.7 $$\int \frac{x\,dx}{(x^3+a^3)^2} = \frac{x^2}{3a^3(x^3+a^3)} + \frac{1}{18a^4}\ln\frac{x^2-ax+a^2}{(x+a)^2} + \frac{1}{3a^4\sqrt{3}}\operatorname{tg}^{-1}\frac{2x-a}{a\sqrt{3}}$$

1.14.8 $$\int \frac{x^2\,dx}{(x^3+a^3)^2} = -\frac{1}{3(x^3+a^3)}$$

1.14.9 $$\int \frac{dx}{x(x^3+a^3)^2} = \frac{1}{3a^3(x^3+a^3)} + \frac{1}{3a^6}\ln\left(\frac{x^3}{x^3+a^3}\right)$$

1.14.10 $$\int \frac{dx}{x^2(x^3+a^3)^2} = -\frac{1}{a^6x} - \frac{x^2}{3a^6(x^3+a^3)} - \frac{4}{3a^6}\int \frac{x\,dx}{x^3+a^3}$$ [Véase 14.2]

1.14.11 $$\int \frac{x^m\,dx}{x^3+a^3} = \frac{x^{m-2}}{m-2} - a^3\int \frac{x^{m-3}\,dx}{x^3+a^3}$$

1.14.12 $$\int \frac{dx}{x^n(x^3+a^3)} = \frac{-1}{a^3(n-1)x^{n-1}} - \frac{1}{a^3}\int \frac{dx}{x^{n-3}(x^3+a^3)}$$

(15) INTEGRALES QUE CONTIENEN $x^4 \pm a^4$

1.15.1 $$\int \frac{dx}{x^4 + a^4} = \frac{1}{4a^3\sqrt{2}} \ln\left(\frac{x^2 + ax\sqrt{2} + a^2}{x^2 - ax\sqrt{2} + a^2}\right) - \frac{1}{2a^3\sqrt{2}}\left[\operatorname{tg}^{-1}\left(1 - \frac{x\sqrt{2}}{a}\right) - \operatorname{tg}^{-1}\left(1 + \frac{x\sqrt{2}}{a}\right)\right]$$

1.15.2 $$\int \frac{x\,dx}{x^4 + a^4} = \frac{1}{2a^2} \operatorname{tg}^{-1} \frac{x^2}{a^2}$$

1.15.3 $$\int \frac{x^2\,dx}{x^4 + a^4} = \frac{1}{4a\sqrt{2}} \ln\left(\frac{x^2 - ax\sqrt{2} + a^2}{x^2 + ax\sqrt{2} + a^2}\right) - \frac{1}{2a\sqrt{2}}\left[\operatorname{tg}^{-1}\left(1 - \frac{x\sqrt{2}}{a}\right) - \operatorname{tg}^{-1}\left(1 + \frac{x\sqrt{2}}{a}\right)\right]$$

1.15.4 $$\int \frac{x^3\,dx}{x^4 + a^4} = \frac{1}{4} \ln(x^4 + a^4)$$

1.15.5 $$\int \frac{dx}{x(x^4 + a^4)} = \frac{1}{4a^4} \ln\left(\frac{x^4}{x^4 + a^4}\right)$$

1.15.6 $$\int \frac{dx}{x^2(x^4 + a^4)} = -\frac{1}{a^4 x} - \frac{1}{4a^5\sqrt{2}} \ln\left(\frac{x^2 - ax\sqrt{2} + a^2}{x^2 + ax\sqrt{2} + a^2}\right) + \frac{1}{2a^5\sqrt{2}}\left[\operatorname{tg}^{-1}\left(1 - \frac{x\sqrt{2}}{a}\right) - \operatorname{tg}^{-1}\left(1 + \frac{x\sqrt{2}}{a}\right)\right]$$

1.15.7 $$\int \frac{dx}{x^3(x^4 + a^4)} = -\frac{1}{2a^4x^2} - \frac{1}{2a^6} \operatorname{tg}^{-1} \frac{x^2}{a^2}$$

1.15.8 $$\int \frac{dx}{x^4 - a^4} = \frac{1}{4a^3} \ln\left(\frac{x - a}{x + a}\right) - \frac{1}{2a^3} \operatorname{tg}^{-1} \frac{x}{a}$$

1.15.9 $$\int \frac{x\,dx}{x^4 - a^4} = \frac{1}{4a^2} \ln\left(\frac{x^2 - a^2}{x^2 + a^2}\right)$$

1.15.10 $$\int \frac{x^2\,dx}{x^4 - a^4} = \frac{1}{4a} \ln\left(\frac{x - a}{x + a}\right) + \frac{1}{2a} \operatorname{tg}^{-1} \frac{x}{a}$$

1.15.11 $$\int \frac{x^3\,dx}{x^4 - a^4} = \frac{1}{4} \ln(x^4 - a^4)$$

1.15.12 $$\int \frac{dx}{x(x^4 - a^4)} = \frac{1}{4a^4} \ln\left(\frac{x^4 - a^4}{x^4}\right)$$

1.15.13 $$\int \frac{dx}{x^3(x^4 - a^4)} = \frac{1}{a^4 x} + \frac{1}{4a^5} \ln\left(\frac{x - a}{x + a}\right) + \frac{1}{2a^5} \operatorname{tg}^{-1} \frac{x}{a}$$

1.15.14 $$\int \frac{dx}{x^3(x^4 - a^4)} = \frac{1}{2a^4x^2} + \frac{1}{4a^6} \ln\left(\frac{x^2 - a^2}{x^2 + a^2}\right)$$

(16) INTEGRALES QUE CONTIENEN $x^n \pm a^n$

1.16.1 $$\int \frac{dx}{x(x^n + a^n)} = \frac{1}{na^n} \ln \frac{x^n}{x^n + a^n}$$

1.16.2 $$\int \frac{x^{n-1}\, dx}{x^n + a^n} = \frac{1}{n} \ln (x^n + a^n)$$

1.16.3 $$\int \frac{x^m\, dx}{(x^n + a^n)^r} = \int \frac{x^{m-n}\, dx}{(x^n + a^n)^{r-1}} - a^n \int \frac{x^{m-n}\, dx}{(x^n + a^n)^r}$$

1.16.4 $$\int \frac{dx}{x^m(x^n + a^n)^r} = \frac{1}{a^n} \int \frac{dx}{x^m(x^n + a^n)^{r-1}} - \frac{1}{a^n} \int \frac{dx}{x^{m-n}(x^n + a^n)^r}$$

1.16.5 $$\int \frac{dx}{x\sqrt{x^n + a^n}} = \frac{1}{n\sqrt{a^n}} \ln \left(\frac{\sqrt{x^n + a^n} - \sqrt{a^n}}{\sqrt{x^n + a^n} + \sqrt{a^n}} \right)$$

1.16.6 $$\int \frac{dx}{x(x^n - a^n)} = \frac{1}{na^n} \ln \left(\frac{x^n - a^n}{x^n} \right)$$

1.16.7 $$\int \frac{x^{n-1}\, dx}{x^n - a^n} = \frac{1}{n} \ln (x^n - a^n)$$

1.16.8 $$\int \frac{x^m\, dx}{(x^n - a^n)^r} = a^n \int \frac{x^{m-n}\, dx}{(x^n - a^n)^r} + \int \frac{x^{m-n}\, dx}{(x^n - a^n)^{r-1}}$$

1.16.9 $$\int \frac{dx}{x^m(x^n - a^n)^r} = \frac{1}{a^n} \int \frac{dx}{x^{m-n}(x^n - a^n)^r} - \frac{1}{a^n} \int \frac{dx}{x^m(x^n - a^n)^{r-1}}$$

1.16.10 $$\int \frac{dx}{x\sqrt{x^n - a^n}} = \frac{2}{n\sqrt{a^n}} \cos^{-1} \sqrt{\frac{a^n}{x^n}}$$

1.16.11 $$\int \frac{x^{p-1}\, dx}{x^{2m} + a^{2m}} = \frac{1}{ma^{2m-p}} \sum_{k=1}^{m} \operatorname{sen} \frac{(2k-1)p\pi}{2m} \operatorname{tg}^{-1} \left(\frac{x + a\cos[(2k-1)\pi/2m]}{a \operatorname{sen}[(2k-1)\pi/2m]} \right)$$
$$- \frac{1}{2ma^{2m-p}} \sum_{k=1}^{m} \cos \frac{(2k-1)p\pi}{2m} \ln \left(x^2 + 2ax \cos \frac{(2k-1)\pi}{2m} + a^2 \right)$$

donde $0 < p \leqq 2m$.

1.16.12 $$\int \frac{x^{p-1}\, dx}{x^{2m} - a^{2m}} = \frac{1}{2ma^{2m-p}} \sum_{k=1}^{m-1} \cos \frac{kp\pi}{m} \ln \left(x^2 - 2ax \cos \frac{k\pi}{m} + a^2 \right)$$
$$- \frac{1}{ma^{2m-p}} \sum_{k=1}^{m-1} \operatorname{sen} \frac{kp\pi}{m} \operatorname{tg}^{-1} \left(\frac{x - a\cos(k\pi/m)}{a \operatorname{sen}(k\pi/m)} \right)$$
$$+ \frac{1}{2ma^{2m-p}} \{ \ln(x - a) + (-1)^p \ln(x + a) \}$$

donde $0 < p \leqq 2m$.

1.16.13 $$\int \frac{x^{p-1}\,dx}{x^{2m+1}+a^{2m+1}} = \frac{2(-1)^{p-1}}{(2m+1)a^{2m-p+1}} \sum_{k=1}^{m} \operatorname{sen} \frac{2kp\pi}{2m+1} \operatorname{tg}^{-1}\left(\frac{x+a\cos[2k\pi/2m+1)]}{a\operatorname{sen}[2k\pi/(2m+1)]}\right)$$
$$-\frac{(-1)^{p-1}}{(2m+1)a^{2m-p+1}} \sum_{k=1}^{m} \cos \frac{2kp\pi}{2m+1} \ln\left(x^2+2ax\cos\frac{2k\pi}{2m+1}+a^2\right)$$
$$+\frac{(-1)^{p-1}\ln(x+a)}{(2m+1)a^{2m-p+1}}$$

donde $0 < p \leqq 2m+1$.

1.16.14 $$\int \frac{x^{p-1}\,dx}{x^{2m+1}-a^{2m+1}} = \frac{-2}{(2m-1)a^{2m-p+1}} \sum_{k=1}^{m} \operatorname{sen} \frac{2kp\pi}{2m+1} \operatorname{tg}^{-1}\left(\frac{x-a\cos[2k\pi/(2m+1)]}{a\operatorname{sen}[2k\pi/(2m+1)]}\right)$$
$$+\frac{1}{(2m+1)a^{2m-p+1}} \sum_{k=1}^{m} \cos \frac{2kp\pi}{2m+1} \ln\left(x^2-2ax\cos\frac{2k\pi}{2m+1}+a^2\right)$$
$$+\frac{\ln(x-a)}{(2m+1)a^{2m-p+1}}$$

donde $0 < p \leqq 2m+1$.

(17) INTEGRALES QUE CONTIENEN sen ax

1.17.1 $$\int \operatorname{sen} ax\,dx = -\frac{\cos ax}{a}$$

1.17.2 $$\int x \operatorname{sen} ax\,dx = \frac{\operatorname{sen} ax}{a^2} - \frac{x\cos ax}{a}$$

1.17.3 $$\int x^2 \operatorname{sen} ax\,dx = \frac{2x}{a^2}\operatorname{sen} ax + \left(\frac{2}{a^3}-\frac{x^2}{a}\right)\cos ax$$

1.17.4 $$\int x^3 \operatorname{sen} ax\,dx = \left(\frac{3x^2}{a^2}-\frac{6}{a^4}\right)\operatorname{sen} ax + \left(\frac{6x}{a^3}-\frac{x^3}{a}\right)\cos ax$$

1.17.5 $$\int \frac{\operatorname{sen} ax}{x}\,dx = ax - \frac{(ax)^3}{3\cdot 3!} + \frac{(ax)^5}{5\cdot 5!} - \cdots$$

1.17.6 $$\int \frac{\operatorname{sen} ax}{x^2}\,dx = -\frac{\operatorname{sen} ax}{x} + a\int \frac{\cos ax}{x}\,dx$$ [Véase 1.18.5].

1.17.7 $$\int \frac{dx}{\operatorname{sen} ax} = \frac{1}{a}\ln(\csc ax - \cot ax) = \frac{1}{a}\ln \operatorname{tg}\frac{ax}{2}$$

1.17.8 $$\int \frac{x\,dx}{\operatorname{sen} ax} = \frac{1}{a^2}\left\{ax + \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \cdots + \frac{2(2^{2n-1}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$$

1.17.9 $$\int \operatorname{sen}^2 ax\,dx = \frac{x}{2} - \frac{\operatorname{sen} 2ax}{4a}$$

1.17.10 $\int x \operatorname{sen}^2 ax\, dx = \dfrac{x^2}{4} - \dfrac{x \operatorname{sen} 2ax}{4a} - \dfrac{\cos 2ax}{8a^2}$

1.17.11 $\int \operatorname{sen}^3 ax\, dx = -\dfrac{\cos ax}{a} + \dfrac{\cos^3 ax}{3a}$

1.17.12 $\int \operatorname{sen}^4 ax\, dx = \dfrac{3x}{8} - \dfrac{\operatorname{sen} 2ax}{4a} + \dfrac{\operatorname{sen} 4ax}{32a}$

1.17.13 $\int \dfrac{dx}{\operatorname{sen}^2 ax} = -\dfrac{1}{a} \cot ax$

1.17.14 $\int \dfrac{dx}{\operatorname{sen}^3 ax} = -\dfrac{\cos ax}{2a \operatorname{sen}^2 ax} + \dfrac{1}{2a} \ln \operatorname{tg} \dfrac{ax}{2}$

1.17.15 $\int \operatorname{sen} px \operatorname{sen} qx\, dx = \dfrac{\operatorname{sen}(p-q)x}{2(p-q)} - \dfrac{\operatorname{sen}(p+q)x}{2(p+q)}$ [Si $p = \pm q$, véase 1.17.9].

1.17.16 $\int \dfrac{dx}{1 - \operatorname{sen} ax} = \dfrac{1}{a} \operatorname{tg}\left(\dfrac{\pi}{4} + \dfrac{ax}{2}\right)$

1.17.17 $\int \dfrac{x\, dx}{1 - \operatorname{sen} ax} = \dfrac{x}{a} \operatorname{tg}\left(\dfrac{\pi}{4} + \dfrac{ax}{2}\right) + \dfrac{2}{a^2} \ln \operatorname{sen}\left(\dfrac{\pi}{4} - \dfrac{ax}{2}\right)$

1.17.18 $\int \dfrac{dx}{1 + \operatorname{sen} ax} = -\dfrac{1}{a} \operatorname{tg}\left(\dfrac{\pi}{4} - \dfrac{ax}{2}\right)$

1.17.19 $\int \dfrac{x\, dx}{1 + \operatorname{sen} ax} = -\dfrac{x}{a} \operatorname{tg}\left(\dfrac{\pi}{4} - \dfrac{ax}{2}\right) + \dfrac{2}{a^2} \ln \operatorname{sen}\left(\dfrac{\pi}{4} + \dfrac{ax}{2}\right)$

1.17.20 $\int \dfrac{dx}{(1 - \operatorname{sen} ax)^2} = \dfrac{1}{2a} \operatorname{tg}\left(\dfrac{\pi}{4} + \dfrac{ax}{2}\right) + \dfrac{1}{6a} \operatorname{tg}^3\left(\dfrac{\pi}{4} + \dfrac{ax}{2}\right)$

1.17.21 $\int \dfrac{dx}{(1 + \operatorname{sen} ax)^2} = -\dfrac{1}{2a} \operatorname{tg}\left(\dfrac{\pi}{4} - \dfrac{ax}{2}\right) - \dfrac{1}{6a} \operatorname{tg}^3\left(\dfrac{\pi}{4} - \dfrac{ax}{2}\right)$

1.17.22 $\int \dfrac{dx}{p + q \operatorname{sen} ax} = \begin{cases} \dfrac{2}{a\sqrt{p^2 - q^2}} \operatorname{tg}^{-1} \dfrac{p \operatorname{tg} \frac{1}{2} ax + q}{\sqrt{p^2 - q^2}} \\ \dfrac{1}{a\sqrt{q^2 - p^2}} \ln\left(\dfrac{p \operatorname{tg} \frac{1}{2} ax + q - \sqrt{q^2 - p^2}}{p \operatorname{tg} \frac{1}{2} ax + q + \sqrt{q^2 - p^2}}\right) \end{cases}$

Si $p = \pm q$, véase 1.17.16 y 1.17.18.

1.17.23 $\int \dfrac{dx}{(p + q \operatorname{sen} ax)^2} = \dfrac{q \cos ax}{a(p^2 - q^2)(p + q \operatorname{sen} ax)} + \dfrac{p}{p^2 - q^2} \int \dfrac{dx}{p + q \operatorname{sen} ax}$

Si $p = \pm q$, véase 1.17.20 y 1.17.21.

1.17.24 $\displaystyle\int \frac{dx}{p^2 + q^2 \operatorname{sen}^2 ax} = \frac{1}{ap\sqrt{p^2 + q^2}} \operatorname{tg}^{-1} \frac{\sqrt{p^2 + q^2}\operatorname{tg} ax}{p}$

1.17.25 $\displaystyle\int \frac{dx}{p^2 - q^2 \operatorname{sen}^2 ax} = \begin{cases} \dfrac{1}{ap\sqrt{p^2 - q^2}} \operatorname{tg}^{-1} \dfrac{\sqrt{p^2 - q^2}\operatorname{tg} ax}{p} \\ \dfrac{1}{2ap\sqrt{q^2 - p^2}} \ln\left(\dfrac{\sqrt{q^2 - p^2}\operatorname{tg} ax + p}{\sqrt{q^2 - p^2}\operatorname{tg} ax - p}\right) \end{cases}$

1.17.26 $\displaystyle\int x^m \operatorname{sen} ax\, dx = -\frac{x^m \cos ax}{a} + \frac{mx^{m-1}\operatorname{sen} ax}{a^2} - \frac{m(m-1)}{a^2}\int x^{m-2}\operatorname{sen} ax\, dx$

1.17.27 $\displaystyle\int \frac{\operatorname{sen} ax}{x^n}dx = -\frac{\operatorname{sen} ax}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{\cos ax}{x^{n-1}}dx$ [Véase 1.18.30].

1.17.28 $\displaystyle\int \operatorname{sen}^n ax\, dx = -\frac{\operatorname{sen}^{n-1} ax \cos ax}{an} + \frac{n-1}{n}\int \operatorname{sen}^{n-2} ax\, dx$

1.17.29 $\displaystyle\int \frac{dx}{\operatorname{sen}^n ax} = \frac{-\cos ax}{a(n-1)\operatorname{sen}^{n-1} ax} + \frac{n-2}{n-1}\int \frac{dx}{\operatorname{sen}^{n-2} ax}$

1.17.30 $\displaystyle\int \frac{x\,dx}{\operatorname{sen}^n ax} = \frac{-x\cos ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\operatorname{sen}^{n-2} ax} + \frac{n-2}{n-1}\int \frac{x\,dx}{\operatorname{sen}^{n-2} ax}$

(18) INTEGRALES QUE CONTIENEN $\cos ax$

1.18.1 $\displaystyle\int \cos ax\, dx = \frac{\operatorname{sen} ax}{a}$

1.18.2 $\displaystyle\int x\cos ax\, dx = \frac{\cos ax}{a^2} + \frac{x \operatorname{sen} ax}{a}$

1.18.3 $\displaystyle\int x^2\cos ax\, dx = \frac{2x}{a^2}\cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3}\right)\operatorname{sen} ax$

1.18.4 $\displaystyle\int x^3\cos ax\, dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4}\right)\cos ax + \left(\frac{x^3}{a} - \frac{6x}{a^3}\right)\operatorname{sen} ax$

1.18.5 $\displaystyle\int \frac{\cos ax}{x}dx = \ln x - \frac{(ax)^2}{2\cdot 2!} + \frac{(ax)^4}{4\cdot 4!} - \frac{(ax)^6}{6\cdot 6!} + \cdots$

1.18.6 $\displaystyle\int \frac{\cos ax}{x^2}dx = -\frac{\cos ax}{x} - a\int \frac{\operatorname{sen} ax}{x}dx$ [Véase 1.17.5].

1.18.7 $\displaystyle\int \frac{dx}{\cos ax} = \frac{1}{a}\ln(\sec ax + \operatorname{tg} ax) = \frac{1}{a}\ln \operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right)$

1.18.8 $$\int \frac{x\,dx}{\cos ax} = \frac{1}{a^2}\left\{\frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \cdots + \frac{E_n(ax)^{2n+2}}{(2n+2)(2n)!} + \cdots\right\}$$

1.18.9 $$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\operatorname{sen} 2ax}{4a}$$

1.18.10 $$\int x\cos^2 ax\,dx = \frac{x^2}{4} + \frac{x \operatorname{sen} 2ax}{4a} + \frac{\cos 2ax}{8a^2}$$

1.18.11 $$\int \cos^3 ax\,dx = \frac{\operatorname{sen} ax}{a} - \frac{\operatorname{sen}^3 ax}{3a}$$

1.18.12 $$\int \cos^4 ax\,dx = \frac{3x}{8} + \frac{\operatorname{sen} 2ax}{4a} + \frac{\operatorname{sen} 4ax}{32a}$$

1.18.13 $$\int \frac{dx}{\cos^2 ax} = \frac{\operatorname{tg} ax}{a}$$

1.18.14 $$\int \frac{dx}{\cos^3 ax} = \frac{\operatorname{sen} ax}{2a\cos^2 ax} + \frac{1}{2a}\ln \operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right)$$

1.18.15 $$\int \cos ax \cos px\,dx = \frac{\operatorname{sen}(a-p)x}{2(a-p)} + \frac{\operatorname{sen}(a+p)x}{2(a+p)}$$ [Si $a = \pm p$, véase 1.18.9].

1.18.16 $$\int \frac{dx}{1-\cos ax} = -\frac{1}{a}\cot\frac{ax}{2}$$

1.18.17 $$\int \frac{x\,dx}{1-\cos ax} = -\frac{x}{a}\cot\frac{ax}{2} + \frac{2}{a^2}\ln \operatorname{sen}\frac{ax}{2}$$

1.18.18 $$\int \frac{dx}{1+\cos ax} = \frac{1}{a}\operatorname{tg}\frac{ax}{2}$$

1.18.19 $$\int \frac{x\,dx}{1+\cos ax} = \frac{x}{a}\operatorname{tg}\frac{ax}{2} + \frac{2}{a^2}\ln\cos\frac{ax}{2}$$

1.18.20 $$\int \frac{dx}{(1-\cos ax)^2} = -\frac{1}{2a}\cot\frac{ax}{2} - \frac{1}{6a}\cot^3\frac{ax}{2}$$

1.18.21 $$\int \frac{dx}{(1+\cos ax)^2} = \frac{1}{2a}\operatorname{tg}\frac{ax}{2} + \frac{1}{6a}\operatorname{tg}^3\frac{ax}{2}$$

1.18.22 $$\int \frac{dx}{p+q\cos ax} = \begin{cases} \dfrac{2}{a\sqrt{p^2-q^2}}\operatorname{tg}^{-1}\sqrt{(p-q)/(p+q)}\operatorname{tg}\tfrac{1}{2}ax \\[2ex] \dfrac{1}{a\sqrt{q^2-p^2}}\ln\left(\dfrac{\operatorname{tg}\frac{1}{2}ax + \sqrt{(q+p)/(q-p)}}{\operatorname{tg}\frac{1}{2}ax - \sqrt{(q+p)/(q-p)}}\right)\end{cases}$$ [Si $p = \pm q$, véanse 1.18.16 y 1.18.18].

1.18.23 $$\int \frac{dx}{(p+q\cos ax)^2} = \frac{q\operatorname{sen} ax}{a(q^2-p^2)(p+q\cos ax)} - \frac{p}{q^2-p^2}\int\frac{dx}{p+q\cos ax}$$ [Si $p = \pm q$, véanse 1.18.19 y 1.18.20].

1.18.24 $$\int \frac{dx}{p^2 + q^2 \cos^2 ax} = \frac{1}{ap\sqrt{p^2+q^2}} \operatorname{tg}^{-1} \frac{p \operatorname{tg} ax}{\sqrt{p^2+q^2}}$$

1.18.25 $$\int \frac{dx}{p^2 - q^2 \cos^2 ax} = \begin{cases} \dfrac{1}{ap\sqrt{p^2-q^2}} \operatorname{tg}^{-1} \dfrac{p \operatorname{tg} ax}{\sqrt{p^2-q^2}} \\ \dfrac{1}{2ap\sqrt{q^2-p^2}} \ln\left(\dfrac{p \operatorname{tg} ax - \sqrt{q^2-p^2}}{p \operatorname{tg} ax + \sqrt{q^2-p^2}}\right) \end{cases}$$

1.18.26 $$\int x^m \cos ax\, dx = \frac{x^m \operatorname{sen} ax}{a} + \frac{mx^{m-1}}{a^2} \cos ax - \frac{m(m-1)}{a^2} \int x^{m-2} \cos ax\, dx$$

1.18.27 $$\int \frac{\cos ax}{x^n} dx = -\frac{\cos ax}{(n-1)x^{n-1}} - \frac{a}{n-1} \int \frac{\operatorname{sen} ax}{x^{n-1}} dx$$ [Véase 1.17.27].

1.18.28 $$\int \cos^n ax\, dx = \frac{\operatorname{sen} ax \cos^{n-1} ax}{an} + \frac{n-1}{n} \int \cos^{n-2} ax\, dx$$

1.18.29 $$\int \frac{dx}{\cos^n ax} = \frac{\operatorname{sen} ax}{a(n-1)\cos^{n-1} ax} + \frac{n-2}{n-1} \int \frac{dx}{\cos^{n-2} ax}$$

1.18.30 $$\int \frac{x\, dx}{\cos^n ax} = \frac{x \operatorname{sen} ax}{a(n-1)\cos^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\cos^{n-2} ax} + \frac{n-2}{n-1} \int \frac{x\, dx}{\cos^{n-2} ax}$$

(19) INTEGRALES QUE CONTIENEN sen ax Y cos ax

1.19.1 $$\int \operatorname{sen} ax \cos ax\, dx = \frac{\operatorname{sen}^2 ax}{2a}$$

1.19.2 $$\int \operatorname{sen} px \cos qx\, dx = -\frac{\cos (p-q)x}{2(p-q)} - \frac{\cos (p+q)x}{2(p+q)}$$

1.19.3 $$\int \operatorname{sen}^n ax \cos ax\, dx = \frac{\operatorname{sen}^{n+1} ax}{(n+1)a}$$ [Si $n = -1$, véase 1.21.1].

1.19.4 $$\int \cos^n ax \operatorname{sen} ax\, dx = -\frac{\cos^{n+1} ax}{(n+1)a}$$ [Si $n = -1$, véase 1.20.1].

1.19.5 $$\int \operatorname{sen}^2 ax \cos^2 ax\, dx = \frac{x}{8} - \frac{\operatorname{sen} 4ax}{32a}$$

1.19.6 $$\int \frac{dx}{\operatorname{sen} ax \cos ax} = \frac{1}{a} \ln \operatorname{tg} ax$$

1.19.7 $$\int \frac{dx}{\operatorname{sen}^2 ax \cos ax} = \frac{1}{a} \ln \operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right) - \frac{1}{a \operatorname{sen} ax}$$

1.19.8 $$\int \frac{dx}{\operatorname{sen} ax \cos^2 ax} = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2} + \frac{1}{a \cos ax}$$

1.19.9 $$\int \frac{dx}{\operatorname{sen}^2 ax \cos^2 ax} = -\frac{2\cot 2ax}{a}$$

1.19.10 $$\int \frac{\operatorname{sen}^2 ax}{\cos ax}\,dx = -\frac{\operatorname{sen} ax}{a} + \frac{1}{a}\ln \operatorname{tg}\left(\frac{ax}{2} + \frac{\pi}{4}\right)$$

1.19.11 $$\int \frac{\cos^2 ax}{\operatorname{sen} ax}\,dx = \frac{\cos ax}{a} + \frac{1}{a}\ln \operatorname{tg}\frac{ax}{2}$$

1.19.12 $$\int \frac{dx}{\cos ax(1 \pm \operatorname{sen} ax)} = \mp \frac{1}{2a(1 \pm \operatorname{sen} ax)} + \frac{1}{2a}\ln \operatorname{tg}\left(\frac{ax}{2} + \frac{\pi}{4}\right)$$

1.19.13 $$\int \frac{dx}{\operatorname{sen} ax(1 \pm \cos ax)} = \pm \frac{1}{2a(1 \pm \cos ax)} + \frac{1}{2a}\ln \operatorname{tg}\frac{ax}{2}$$

1.19.14 $$\int \frac{dx}{\operatorname{sen} ax \pm \cos ax} = \frac{1}{a\sqrt{2}}\ln \operatorname{tg}\left(\frac{ax}{2} \pm \frac{\pi}{8}\right)$$

1.19.15 $$\int \frac{\operatorname{sen} ax\,dx}{\operatorname{sen} ax \pm \cos ax} = \frac{x}{2} \mp \frac{1}{2a}\ln(\operatorname{sen} ax \pm \cos ax)$$

1.19.16 $$\int \frac{\cos ax\,dx}{\operatorname{sen} ax \pm \cos ax} = \pm\frac{x}{2} + \frac{1}{2a}\ln(\operatorname{sen} ax \pm \cos ax)$$

1.19.17 $$\int \frac{\operatorname{sen} ax\,dx}{p + q\cos ax} = -\frac{1}{aq}\ln(p + q\cos ax)$$

1.19.18 $$\int \frac{\cos ax\,dx}{p + q\operatorname{sen} ax} = \frac{1}{aq}\ln(p + q\operatorname{sen} ax)$$

1.19.19 $$\int \frac{\operatorname{sen} ax\,dx}{(p + q\cos ax)^n} = \frac{1}{aq(n-1)(p + q\cos ax)^{n-1}}$$

1.19.20 $$\int \frac{\cos ax\,dx}{(p + q\operatorname{sen} ax)^n} = \frac{-1}{aq(n-1)(p + q\operatorname{sen} ax)^{n-1}}$$

1.19.21 $$\int \frac{dx}{p\operatorname{sen} ax + q\cos ax} = \frac{1}{a\sqrt{p^2+q^2}}\ln \operatorname{tg}\left(\frac{ax + \operatorname{tg}^{-1}(q/p)}{2}\right)$$

1.19.22 $$\int \frac{dx}{p\operatorname{sen} ax + q\cos ax + r} = \begin{cases} \dfrac{2}{a\sqrt{r^2-p^2-q^2}}\operatorname{tg}^{-1}\left(\dfrac{p + (r-q)\operatorname{tg}(ax/2)}{\sqrt{r^2-p^2-q^2}}\right) \\[2ex] \dfrac{1}{a\sqrt{p^2+q^2-r^2}}\ln\left(\dfrac{p - \sqrt{p^2+q^2-r^2} + (r-q)\operatorname{tg}(ax/2)}{p + \sqrt{p^2+q^2-r^2} + (r-q)\operatorname{tg}(ax/2)}\right) \end{cases}$$

Si $r = q$, véase 1.19.23. Si $r^2 = p^2 + q^2$, véase 1.19.24.

1.19.23 $$\int \frac{dx}{p\operatorname{sen} ax + q(1 + \cos ax)} = \frac{1}{ap}\ln\left(q + p\operatorname{tg}\frac{ax}{2}\right)$$

1.19.24 $$\int \frac{dx}{p\operatorname{sen} ax + q\cos ax \pm \sqrt{p^2+q^2}} = \frac{-1}{a\sqrt{p^2+q^2}}\operatorname{tg}\left(\frac{\pi}{4} \mp \frac{ax + \operatorname{tg}^{-1}(q/p)}{2}\right)$$

1.19.25 $\displaystyle\int \frac{dx}{p^2 \operatorname{sen}^2 ax + q^2 \cos^2 ax} = \frac{1}{apq} \operatorname{tg}^{-1}\left(\frac{p \operatorname{tg} ax}{q}\right)$

1.19.26 $\displaystyle\int \frac{dx}{p^2 \operatorname{sen}^2 ax - q^2 \cos^2 ax} = \frac{1}{2apq} \ln\left(\frac{p \operatorname{tg} ax - q}{p \operatorname{tg} ax + q}\right)$

1.19.27 $$\int \operatorname{sen}^m ax \cos^n ax\, dx = \begin{cases} -\dfrac{\operatorname{sen}^{m-1} ax \cos^{n+1} ax}{a(m+n)} + \dfrac{m-1}{m+n} \displaystyle\int \operatorname{sen}^{m-2} ax \cos^n ax\, dx \\ \dfrac{\operatorname{sen}^{m+1} ax \cos^{n-1} ax}{a(m+n)} + \dfrac{n-1}{m+n} \displaystyle\int \operatorname{sen}^m ax \cos^{n-2} ax\, dx \end{cases}$$

1.19.28 $$\int \frac{\operatorname{sen}^m ax}{\cos^n ax} dx = \begin{cases} \dfrac{\operatorname{sen}^{m-1} ax}{a(n-1)\cos^{n-1} ax} - \dfrac{m-1}{n-1} \displaystyle\int \frac{\operatorname{sen}^{m-2} ax}{\cos^{n-2} ax} dx \\ \dfrac{\operatorname{sen}^{m+1} ax}{a(n-1)\cos^{n-1} ax} - \dfrac{m-n+2}{n-1} \displaystyle\int \frac{\operatorname{sen}^m ax}{\cos^{n-2} ax} dx \\ \dfrac{-\operatorname{sen}^{m-1} ax}{a(m-n)\cos^{n-1} ax} + \dfrac{m-1}{m-n} \displaystyle\int \frac{\operatorname{sen}^{m-2} ax}{\cos^n ax} dx \end{cases}$$

1.19.29 $$\int \frac{\cos^m ax}{\operatorname{sen}^n ax} dx = \begin{cases} \dfrac{-\cos^{m-1} ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \dfrac{m-1}{n-1} \displaystyle\int \frac{\cos^{m-2} ax}{\operatorname{sen}^{n-2} ax} dx \\ \dfrac{-\cos^{m+1} ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \dfrac{m-n+2}{n-1} \displaystyle\int \frac{\cos^m ax}{\operatorname{sen}^{n-2} ax} dx \\ \dfrac{\cos^{m-1} ax}{a(m-n)\operatorname{sen}^{n-1} ax} + \dfrac{m-1}{m-n} \displaystyle\int \frac{\cos^{m-2} ax}{\operatorname{sen}^n ax} dx \end{cases}$$

1.19.30 $$\int \frac{dx}{\operatorname{sen}^m ax \cos^n ax} = \begin{cases} \dfrac{1}{a(n-1)\operatorname{sen}^{m-1} ax \cos^{n-1} ax} + \dfrac{m+n-2}{n-1} \displaystyle\int \frac{dx}{\operatorname{sen}^m ax \cos^{n-2} ax} \\ \dfrac{-1}{a(m-1)\operatorname{sen}^{m-1} ax \cos^{n-1} ax} + \dfrac{m+n-2}{m-1} \displaystyle\int \frac{dx}{\operatorname{sen}^{m-2} ax \cos^n ax} \end{cases}$$

(20) INTEGRALES QUE CONTIENEN tg *ax*

1.20.1 $\displaystyle\int \operatorname{tg} ax\, dx = -\frac{1}{a} \ln \cos ax = \frac{1}{a} \ln \sec ax$

1.20.2 $\displaystyle\int \operatorname{tg}^2 ax\, dx = \frac{\operatorname{tg} ax}{a} - x$

1.20.3 $\displaystyle\int \operatorname{tg}^3 ax\, dx = \frac{\operatorname{tg}^2 ax}{2a} + \frac{1}{a} \ln \cos ax$

1.20.4 $\displaystyle\int \operatorname{tg}^n ax \sec^2 ax\, dx = \frac{\operatorname{tg}^{n+1} ax}{(n+1)a}$

1.20.5 $$\int \frac{\sec^2 ax}{\operatorname{tg} ax}\,dx = \frac{1}{a}\ln \operatorname{tg} ax$$

1.20.6 $$\int \frac{dx}{\operatorname{tg} ax} = \frac{1}{a}\ln \operatorname{sen} ax$$

1.20.7 $$\int x \operatorname{tg} ax\,dx = \frac{1}{a^2}\left\{\frac{(ax)^3}{3} + \frac{(ax)^5}{15} + \frac{2(ax)^7}{105} + \cdots + \frac{2^{2n}(2^{2n}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$$

1.20.8 $$\int \frac{\operatorname{tg} ax}{x}\,dx = ax + \frac{(ax)^3}{9} + \frac{2(ax)^5}{75} + \cdots + \frac{2^{2n}(2^{2n}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$$

1.20.9 $$\int x \operatorname{tg}^2 ax\,dx = \frac{x \operatorname{tg} ax}{a} + \frac{1}{a^2}\ln \cos ax - \frac{x^2}{2}$$

1.20.10 $$\int \frac{dx}{p + q \operatorname{tg} ax} = \frac{px}{p^2 + q^2} + \frac{q}{a(p^2 + q^2)}\ln(q \operatorname{sen} ax + p \cos ax)$$

1.20.11 $$\int \operatorname{tg}^n ax\,dx = \frac{\operatorname{tg}^{n-1} ax}{(n-1)a} - \int \operatorname{tg}^{n-2} ax\,dx$$

(21) INTEGRALES QUE CONTIENEN $\cot ax$

1.21.1 $$\int \cot ax\,dx = \frac{1}{a}\ln \operatorname{sen} ax$$

1.21.2 $$\int \cot^2 ax\,dx = -\frac{\cot ax}{a} - x$$

1.21.3 $$\int \cot^3 ax\,dx = -\frac{\cot^2 ax}{2a} - \frac{1}{a}\ln \operatorname{sen} ax$$

1.21.4 $$\int \cot^n ax \csc^2 ax\,dx = -\frac{\cot^{n+1} ax}{(n+1)a}$$

1.21.5 $$\int \frac{\csc^2 ax}{\cot ax}\,dx = -\frac{1}{a}\ln \cot ax$$

1.21.6 $$\int \frac{dx}{\cot ax} = -\frac{1}{a}\ln \cos ax$$

1.21.7 $$\int x \cot ax\,dx = \frac{1}{a^2}\left\{ax - \frac{(ax)^3}{9} - \frac{(ax)^5}{225} - \cdots - \frac{2^{2n}B_n(ax)^{2n+1}}{(2n+1)!} - \cdots\right\}$$

1.21.8 $$\int \frac{\cot ax}{x}\,dx = -\frac{1}{ax} - \frac{ax}{3} - \frac{(ax)^3}{135} - \cdots - \frac{2^{2n}B_n(ax)^{2n-1}}{(2n-1)(2n)!} - \cdots$$

1.21.9 $$\int x \cot^2 ax\,dx = -\frac{x \cot ax}{a} + \frac{1}{a^2}\ln \operatorname{sen} ax - \frac{x^2}{2}$$

1.21.10 $$\int \frac{dx}{p + q\cot ax} = \frac{px}{p^2 + q^2} - \frac{q}{a(p^2 + q^2)} \ln(q \operatorname{sen} ax + q \cos ax)$$

1.21.11 $$\int \cot^n ax\, dx = -\frac{\cot^{n-1} ax}{(n-1)a} - \int \cot^{n-2} ax\, dx$$

(22) INTEGRALES QUE CONTIENEN sec *ax*

1.22.1 $$\int \sec ax\, dx = \frac{1}{a} \ln(\sec ax + \operatorname{tg} ax) = \frac{1}{a} \ln \operatorname{tg}\left(\frac{ax}{2} + \frac{\pi}{4}\right)$$

1.22.2 $$\int \sec^2 ax\, dx = \frac{\operatorname{tg} ax}{a}$$

1.22.3 $$\int \sec^3 ax\, dx = \frac{\sec ax \operatorname{tg} ax}{2a} + \frac{1}{2a} \ln(\sec ax + \operatorname{tg} ax)$$

1.22.4 $$\int \sec^n ax \operatorname{tg} ax\, dx = \frac{\sec^n ax}{na}$$

1.22.5 $$\int \frac{dx}{\sec ax} = \frac{\operatorname{sen} ax}{a}$$

1.22.6 $$\int x \sec ax\, dx = \frac{1}{a^2}\left\{\frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \cdots + \frac{E_n(ax)^{2n+2}}{(2n+2)(2n)!} + \cdots\right\}$$

1.22.7 $$\int \frac{\sec ax}{x}\, dx = \ln x + \frac{(ax)^2}{4} + \frac{5(ax)^4}{96} + \frac{61(ax)^6}{4320} + \cdots + \frac{E_n(ax)^{2n}}{2n(2n)!} + \cdots$$

1.22.8 $$\int x \sec^2 ax\, dx = \frac{x}{a} \operatorname{tg} ax + \frac{1}{a^2} \ln \cos ax$$

1.22.9 $$\int \frac{dx}{q + p\sec ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{p + q\cos ax}$$

1.22.10 $$\int \sec^n ax\, dx = \frac{\sec^{n-2} ax \operatorname{tg} ax}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax\, dx$$

(23) INTEGRALES QUE CONTIENEN csc *ax*

1.23.1 $$\int \csc ax\, dx = \frac{1}{a} \ln(\csc ax - \cot ax) = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2}$$

1.23.2 $$\int \csc^2 ax\, dx = -\frac{\cot ax}{a}$$

1.23.3 $$\int \csc^3 ax\,dx = -\frac{\csc ax \cot ax}{2a} + \frac{1}{2a}\ln \operatorname{tg}\frac{ax}{2}$$

1.23.4 $$\int \csc^n ax \cot ax\,dx = -\frac{\csc^n ax}{na}$$

1.23.5 $$\int \frac{dx}{\csc ax} = -\frac{\cos ax}{a}$$

1.23.6 $$\int x\csc ax\,dx = \frac{1}{a^2}\left\{ax + \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \cdots + \frac{2(2^{2n-1}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$$

1.23.7 $$\int \frac{\csc ax}{x}dx = -\frac{1}{ax} + \frac{ax}{6} + \frac{7(ax)^3}{1080} + \cdots + \frac{2(2^{2n-1}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$$

1.23.8 $$\int x\csc^2 ax\,dx = -\frac{x\cot ax}{a} + \frac{1}{a^2}\ln \operatorname{sen} ax$$

1.23.9 $$\int \frac{dx}{q + p\csc ax} = \frac{x}{q} - \frac{p}{q}\int \frac{dx}{p + q \operatorname{sen} ax}$$ [Véase 1.17.22].

1.23.10 $$\int \csc^n ax\,dx = -\frac{\csc^{n-2} ax \cot ax}{a(n-1)} + \frac{n-2}{n-1}\int \csc^{n-2} ax\,dx$$

(24) INTEGRALES QUE CONTIENEN FUNCIONES TRIGONOMÉTRICAS INVERSAS

1.24.1 $$\int \operatorname{sen}^{-1}\frac{x}{a}dx = x\operatorname{sen}^{-1}\frac{x}{a} + \sqrt{a^2-x^2}$$

1.24.2 $$\int x\operatorname{sen}^{-1}\frac{x}{a}dx = \left(\frac{x^2}{2} - \frac{a^2}{4}\right)\operatorname{sen}^{-1}\frac{x}{a} + \frac{x\sqrt{a^2-x^2}}{4}$$

1.24.3 $$\int x^2\operatorname{sen}^{-1}\frac{x}{a}dx = \frac{x^3}{3}\operatorname{sen}^{-1}\frac{x}{a} + \frac{(x^2+2a^2)\sqrt{a^2-x^2}}{9}$$

1.24.4 $$\int \frac{\operatorname{sen}^{-1}(x/a)}{x}dx = \frac{x}{a} + \frac{(x/a)^3}{2\cdot 3\cdot 3} + \frac{1\cdot 3(x/a)^5}{2\cdot 4\cdot 5\cdot 5} + \frac{1\cdot 3\cdot 5(x/a)^7}{2\cdot 4\cdot 6\cdot 7\cdot 7} + \cdots$$

1.24.5 $$\int \frac{\operatorname{sen}^{-1}(x/a)}{x^2}dx = -\frac{\operatorname{sen}^{-1}(x/a)}{x} - \frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

1.24.6 $$\int \left(\operatorname{sen}^{-1}\frac{x}{a}\right)^2 dx = x\left(\operatorname{sen}^{-1}\frac{x}{a}\right)^2 - 2x + 2\sqrt{a^2-x^2}\operatorname{sen}^{-1}\frac{x}{a}$$

1.24.7 $$\int \cos^{-1}\frac{x}{a}dx = x\cos^{-1}\frac{x}{a} - \sqrt{a^2-x^2}$$

1.24.8 $$\int x\cos^{-1}\frac{x}{a}dx = \left(\frac{x^2}{2} - \frac{a^2}{4}\right)\cos^{-1}\frac{x}{a} - \frac{x\sqrt{a^2-x^2}}{4}$$

1.24.9 $\int x^2 \cos^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\cos^{-1}\frac{x}{a} - \frac{(x^2+2a^2)\sqrt{a^2-x^2}}{9}$

1.24.10 $\int \frac{\cos^{-1}(x/a)}{x}\,dx = \frac{\pi}{2}\ln x - \int \frac{\operatorname{sen}^{-1}(x/a)}{x}\,dx$ [Véase 1.24.4].

1.24.11 $\int \frac{\cos^{-1}(x/a)}{x^2}\,dx = -\frac{\cos^{-1}(x/a)}{x} + \frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

1.24.12 $\int \left(\cos^{-1}\frac{x}{a}\right)^2 dx = x\left(\cos^{-1}\frac{x}{a}\right)^2 - 2x - 2\sqrt{a^2-x^2}\cos^{-1}\frac{x}{a}$

1.24.13 $\int \operatorname{tg}^{-1}\frac{x}{a}\,dx = x\operatorname{tg}^{-1}\frac{x}{a} - \frac{a}{2}\ln(x^2+a^2)$

1.24.14 $\int x\operatorname{tg}^{-1}\frac{x}{a}\,dx = \frac{1}{2}(x^2+a^2)\operatorname{tg}^{-1}\frac{x}{a} - \frac{ax}{2}$

1.24.15 $\int x^2\operatorname{tg}^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\operatorname{tg}^{-1}\frac{x}{a} - \frac{ax^2}{6} + \frac{a^3}{6}\ln(x^2+a^2)$

1.24.16 $\int \frac{\operatorname{tg}^{-1}(x/a)}{x}\,dx = \frac{x}{a} - \frac{(x/a)^3}{3^2} + \frac{(x/a)^5}{5^2} - \frac{(x/a)^7}{7^2} + \cdots$

1.24.17 $\int \frac{\operatorname{tg}^{-1}(x/a)}{x^2}\,dx = -\frac{1}{x}\operatorname{tg}^{-1}\frac{x}{a} - \frac{1}{2a}\ln\left(\frac{x^2+a^2}{x^2}\right)$

1.24.18 $\int \cot^{-1}\frac{x}{a}\,dx = x\cot^{-1}\frac{x}{a} + \frac{a}{2}\ln(x^2+a^2)$

1.24.19 $\int x\cot^{-1}\frac{x}{a}\,dx = \frac{1}{2}(x^2+a^2)\cot^{-1}\frac{x}{a} + \frac{ax}{2}$

1.24.20 $\int x^2\cot^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\cot^{-1}\frac{x}{a} + \frac{ax^2}{6} - \frac{a^3}{6}\ln(x^2+a^2)$

1.24.21 $\int \frac{\cot^{-1}(x/a)}{x}\,dx = \frac{\pi}{2}\ln x - \int \frac{\operatorname{tg}^{-1}(x/a)}{x}\,dx$ [Véase 1.24.16].

1.24.22 $\int \frac{\cot^{-1}(x/a)}{x^2}\,dx = -\frac{\cot^{-1}(x/a)}{x} + \frac{1}{2a}\ln\left(\frac{x^2+a^2}{x^2}\right)$

1.24.23 $$\int \sec^{-1}\frac{x}{a}\,dx = \begin{cases} x\sec^{-1}\frac{x}{a} - a\ln(x+\sqrt{x^2-a^2}) & 0 < \sec^{-1}\frac{x}{a} < \frac{\pi}{2} \\ x\sec^{-1}\frac{x}{a} + a\ln(x+\sqrt{x^2-a^2}) & \frac{\pi}{2} < \sec^{-1}\frac{x}{a} < \pi \end{cases}$$

1.24.24 $$\int x \sec^{-1}\frac{x}{a}\,dx = \begin{cases} \dfrac{x^2}{2}\sec^{-1}\dfrac{x}{a} - \dfrac{a\sqrt{x^2-a^2}}{2} & 0 < \sec^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\ \dfrac{x^2}{2}\sec^{-1}\dfrac{x}{a} + \dfrac{a\sqrt{x^2-a^2}}{2} & \dfrac{\pi}{2} < \sec^{-1}\dfrac{x}{a} < \pi \end{cases}$$

1.24.25 $$\int x^2 \sec^{-1}\frac{x}{a}\,dx = \begin{cases} \dfrac{x^3}{3}\sec^{-1}\dfrac{x}{a} - \dfrac{ax\sqrt{x^2-a^2}}{6} - \dfrac{a^3}{6}\ln(x+\sqrt{x^2-a^2}) & 0 < \sec^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\ \dfrac{x^3}{3}\sec^{-1}\dfrac{x}{a} + \dfrac{ax\sqrt{x^2-a^2}}{6} + \dfrac{a^3}{6}\ln(x+\sqrt{x^2-a^2}) & \dfrac{\pi}{2} < \sec^{-1}\dfrac{x}{a} < \pi \end{cases}$$

1.24.26 $$\int \frac{\sec^{-1}(x/a)}{x}\,dx = \frac{\pi}{2}\ln x + \frac{a}{x} + \frac{(a/x)^3}{2\cdot 3\cdot 3} + \frac{1\cdot 3(a/x)^5}{2\cdot 4\cdot 5\cdot 5} + \frac{1\cdot 3\cdot 5(a/x)^7}{2\cdot 4\cdot 6\cdot 7\cdot 7} + \cdots$$

1.24.27 $$\int \frac{\sec^{-1}(x/a)}{x^2}\,dx = \begin{cases} -\dfrac{\sec^{-1}(x/a)}{x} + \dfrac{\sqrt{x^2-a^2}}{ax} & 0 < \sec^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\ -\dfrac{\sec^{-1}(x/a)}{x} - \dfrac{\sqrt{x^2-a^2}}{ax} & \dfrac{\pi}{2} < \sec^{-1}\dfrac{x}{a} < \pi \end{cases}$$

1.24.28 $$\int \csc^{-1}\frac{x}{a}\,dx = \begin{cases} x\csc^{-1}\dfrac{x}{a} + a\ln(x+\sqrt{x^2-a^2}) & 0 < \csc^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\ x\csc^{-1}\dfrac{x}{a} - a\ln(x+\sqrt{x^2-a^2}) & -\dfrac{\pi}{2} < \csc^{-1}\dfrac{x}{a} < 0 \end{cases}$$

1.24.29 $$\int x \csc^{-1}\frac{x}{a}\,dx = \begin{cases} \dfrac{x^2}{2}\csc^{-1}\dfrac{x}{a} + \dfrac{a\sqrt{x^2-a^2}}{2} & 0 < \csc^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\ \dfrac{x^2}{2}\csc^{-1}\dfrac{x}{a} - \dfrac{a\sqrt{x^2-a^2}}{2} & -\dfrac{\pi}{2} < \csc^{-1}\dfrac{x}{a} < 0 \end{cases}$$

1.24.30 $$\int x^2 \csc^{-1}\frac{x}{a}\,dx = \begin{cases} \dfrac{x^3}{3}\csc^{-1}\dfrac{x}{a} + \dfrac{ax\sqrt{x^2-a^2}}{6} + \dfrac{a^3}{6}\ln(x+\sqrt{x^2-a^2}) & 0 < \csc^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\ \dfrac{x^3}{3}\csc^{-1}\dfrac{x}{a} - \dfrac{ax\sqrt{x^2-a^2}}{6} - \dfrac{a^3}{6}\ln(x+\sqrt{x^2-a^2}) & -\dfrac{\pi}{2} < \csc^{-1}\dfrac{x}{a} < 0 \end{cases}$$

1.24.31 $$\int \frac{\csc^{-1}(x/a)}{x}\,dx = -\left(\frac{a}{x} + \frac{(a/x)^3}{2\cdot 3\cdot 3} + \frac{1\cdot 3(a/x)^5}{2\cdot 4\cdot 5\cdot 5} + \frac{1\cdot 3\cdot 5(a/x)^7}{2\cdot 4\cdot 6\cdot 7\cdot 7} + \cdots\right)$$

1.24.32 $$\int \frac{\csc^{-1}(x/a)}{x^2}\,dx = \begin{cases} -\dfrac{\csc^{-1}(x/a)}{x} - \dfrac{\sqrt{x^2-a^2}}{ax} & 0 < \csc^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\ -\dfrac{\csc^{-1}(x/a)}{x} + \dfrac{\sqrt{x^2-a^2}}{ax} & -\dfrac{\pi}{2} < \csc^{-1}\dfrac{x}{a} < 0 \end{cases}$$

1.24.33 $$\int x^m \operatorname{sen}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{sen}^{-1}\frac{x}{a} - \frac{1}{m+1}\int \frac{x^{m+1}}{\sqrt{a^2-x^2}}\,dx$$

1.24.34 $$\int x^m \cos^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\cos^{-1}\frac{x}{a} + \frac{1}{m+1}\int\frac{x^{m+1}}{\sqrt{a^2-x^2}}\,dx$$

1.24.35 $$\int x^m \operatorname{tg}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{tg}^{-1}\frac{x}{a} - \frac{a}{m+1}\int\frac{x^{m+1}}{x^2+a^2}\,dx$$

1.24.36 $$\int x^m \cot^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\cot^{-1}\frac{x}{a} + \frac{a}{m+1}\int\frac{x^{m+1}}{x^2+a^2}\,dx$$

1.24.37 $$\int x^m \sec^{-1}\frac{x}{a}\,dx = \begin{cases} \dfrac{x^{m+1}\sec^{-1}(x/a)}{m+1} - \dfrac{a}{m+1}\displaystyle\int\frac{x^m\,dx}{\sqrt{x^2-a^2}} & 0 < \sec^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\ \dfrac{x^{m+1}\sec^{-1}(x/a)}{m+1} + \dfrac{a}{m+1}\displaystyle\int\frac{x^m\,dx}{\sqrt{x^2-a^2}} & \dfrac{\pi}{2} < \sec^{-1}\dfrac{x}{a} < \pi \end{cases}$$

1.24.38 $$\int x^m \csc^{-1}\frac{x}{a}\,dx = \begin{cases} \dfrac{x^{m+1}\csc^{-1}(x/a)}{m+1} + \dfrac{a}{m+1}\displaystyle\int\frac{x^m\,dx}{\sqrt{x^2-a^2}} & 0 < \csc^{-1}\dfrac{x}{a} < \dfrac{\pi}{2} \\ \dfrac{x^{m+1}\csc^{-1}(x/a)}{m+1} - \dfrac{a}{m+1}\displaystyle\int\frac{x^m\,dx}{\sqrt{x^2-a^2}} & -\dfrac{\pi}{2} < \csc^{-1}\dfrac{x}{a} < 0 \end{cases}$$

(25) INTEGRALES QUE CONTIENEN e^{ax}

1.25.1 $$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

1.25.2 $$\int xe^{ax}\,dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$$

1.25.3 $$\int x^2e^{ax}\,dx = \frac{e^{ax}}{a}\left(x^2 - \frac{2x}{a} + \frac{2}{a^2}\right)$$

1.25.4 $$\int x^ne^{ax}\,dx = \frac{x^ne^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$$
$$= \frac{e^{ax}}{a}\left(x^n - \frac{nx^{n-1}}{a} + \frac{n(n-1)x^{n-2}}{a^2} - \cdots + \frac{(-1)^n n!}{a^n}\right) \quad \text{si } n = \text{entero positivo.}$$

1.25.5 $$\int\frac{e^{ax}}{x}\,dx = \ln x + \frac{ax}{1\cdot 1!} + \frac{(ax)^2}{2\cdot 2!} + \frac{(ax)^3}{3\cdot 3!} + \cdots$$

1.25.6 $$\int\frac{e^{ax}}{x^n}\,dx = \frac{-e^{ax}}{(n-1)x^{n-1}} + \frac{a}{n-1}\int\frac{e^{ax}}{x^{n-1}}\,dx$$

1.25.7 $$\int\frac{dx}{p+qe^{ax}} = \frac{x}{p} - \frac{1}{ap}\ln(p+qe^{ax})$$

1.25.8 $$\int\frac{dx}{(p+qe^{ax})^2} = \frac{x}{p^2} + \frac{1}{ap(p+qe^{ax})} - \frac{1}{ap^2}\ln(p+qe^{ax})$$

1.25.9 $$\int \frac{dx}{pe^{ax} + qe^{-ax}} = \begin{cases} \dfrac{1}{a\sqrt{pq}} \operatorname{tg}^{-1}\left(\sqrt{\dfrac{p}{q}}\, e^{ax}\right) \\ \dfrac{1}{2a\sqrt{-pq}} \ln\left(\dfrac{e^{ax} - \sqrt{-q/p}}{e^{ax} + \sqrt{-q/p}}\right) \end{cases}$$

1.25.10 $$\int e^{ax} \operatorname{sen} bx\, dx = \frac{e^{ax}(a \operatorname{sen} bx - b \cos bx)}{a^2 + b^2}$$

1.25.11 $$\int e^{ax} \cos bx\, dx = \frac{e^{ax}(a \cos bx + b \operatorname{sen} bx)}{a^2 + b^2}$$

1.25.12 $$\int xe^{ax} \operatorname{sen} bx\, dx = \frac{xe^{ax}(a \operatorname{sen} bx - b \cos bx)}{a^2 + b^2} - \frac{e^{ax}\{(a^2 - b^2)\operatorname{sen} bx - 2ab \cos bx\}}{(a^2 + b^2)^2}$$

1.25.13 $$\int xe^{ax} \cos bx\, dx = \frac{xe^{ax}(a \cos bx + b \operatorname{sen} bx)}{a^2 + b^2} - \frac{e^{ax}\{(a^2 - b^2)\cos bx - 2ab \operatorname{sen} bx\}}{(a^2 + b^2)^2}$$

1.25.14 $$\int e^{ax} \ln x\, dx = \frac{e^{ax} \ln x}{a} - \frac{1}{a}\int \frac{e^{ax}}{x}\, dx$$

1.25.15 $$\int e^{ax} \operatorname{sen}^n bx\, dx = \frac{e^{ax} \operatorname{sen}^{n-1} bx}{a^2 + n^2b^2}(a \operatorname{sen} bx - nb \cos bx) + \frac{n(n-1)b^2}{a^2 + n^2b^2}\int e^{ax} \operatorname{sen}^{n-2} bx\, dx$$

1.25.16 $$\int e^{ax} \cos^n bx\, dx = \frac{e^{ax} \cos^{n-1} bx}{a^2 + n^2b^2}(a \cos bx + nb \operatorname{sen} bx) + \frac{n(n-1)b^2}{a^2 + n^2b^2}\int e^{ax} \cos^{n-2} bx\, dx$$

(26) INTEGRALES QUE CONTIENEN $\ln x$

1.26.1 $$\int \ln x\, dx = x \ln x - x$$

1.26.2 $$\int x \ln x\, dx = \frac{x^2}{2}(\ln x - \tfrac{1}{2})$$

1.26.3 $$\int x^m \ln x\, dx = \frac{x^{m+1}}{m+1}\left(\ln x - \frac{1}{m+1}\right)$$ [Si $m = -1$, véase 1.26.4].

1.26.4 $$\int \frac{\ln x}{x}\, dx = \frac{1}{2}\ln^2 x$$

1.26.5 $$\int \frac{\ln x}{x^2}\, dx = -\frac{\ln x}{x} - \frac{1}{x}$$

1.26.6 $$\int \ln^2 x\, dx = x \ln^2 x - 2x \ln x + 2x$$

1.26.7 $$\int \frac{\ln^n x\, dx}{x} = \frac{\ln^{n+1} x}{n+1}$$ [Si $n = -1$, véase 1.26.8].

1.26.8 $\displaystyle\int \frac{dx}{x\ln x} = \ln(\ln x)$

1.26.9 $\displaystyle\int \frac{dx}{\ln x} = \ln(\ln x) + \ln x + \frac{\ln^2 x}{2\cdot 2!} + \frac{\ln^3 x}{3\cdot 3!} + \cdots$

1.26.10 $\displaystyle\int \frac{x^m\,dx}{\ln x} = \ln(\ln x) + (m+1)\ln x + \frac{(m+1)^2\ln^2 x}{2\cdot 2!} + \frac{(m+1)^3\ln^3 x}{3\cdot 3!} + \cdots$

1.26.11 $\displaystyle\int \ln^n x\,dx = x\ln^n x - n\int \ln^{n-1} x\,dx$

1.26.12 $\displaystyle\int x^m \ln^n x\,dx = \frac{x^{m+1}\ln^n x}{m+1} - \frac{n}{m+1}\int x^m \ln^{n-1} x\,dx$ [Si $m = -1$, véase 1.26.7].

1.26.13 $\displaystyle\int \ln(x^2+a^2)\,dx = x\ln(x^2+a^2) - 2x + 2a\,\mathrm{tg}^{-1}\frac{x}{a}$

1.26.14 $\displaystyle\int \ln(x^2-a^2)\,dx = x\ln(x^2-a^2) - 2x + a\ln\left(\frac{x+a}{x-a}\right)$

1.26.15 $\displaystyle\int x^m \ln(x^2\pm a^2)\,dx = \frac{x^{m+1}\ln(x^2\pm a^2)}{m+1} - \frac{2}{m+1}\int \frac{x^{m+2}}{x^2\pm a^2}\,dx$

(27) INTEGRALES QUE CONTIENEN sh ax

1.27.1 $\displaystyle\int \mathrm{sh}\,ax\,dx = \frac{\mathrm{ch}\,ax}{a}$

1.27.2 $\displaystyle\int x\,\mathrm{sh}\,ax\,dx = \frac{x\,\mathrm{ch}\,ax}{a} - \frac{\mathrm{sh}\,ax}{a^2}$

1.27.3 $\displaystyle\int x^2\,\mathrm{sh}\,ax\,dx = \left(\frac{x^2}{a} + \frac{2}{a^3}\right)\mathrm{ch}\,ax - \frac{2x}{a^2}\,\mathrm{sh}\,ax$

1.27.4 $\displaystyle\int \frac{\mathrm{sh}\,ax}{x}\,dx = ax + \frac{(ax)^3}{3\cdot 3!} + \frac{(ax)^5}{5\cdot 5!} + \cdots$

1.27.5 $\displaystyle\int \frac{\mathrm{sh}\,ax}{x^2}\,dx = -\frac{\mathrm{sh}\,ax}{x} + a\int \frac{\mathrm{ch}\,ax}{x}\,dx$ [Véase 1.28.4].

1.27.6 $\displaystyle\int \frac{dx}{\mathrm{sh}\,ax} = \frac{1}{a}\ln\mathrm{th}\,\frac{ax}{2}$

1.27.7 $\displaystyle\int \frac{x\,dx}{\mathrm{sh}\,ax} = \frac{1}{a^2}\left\{ax - \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} - \cdots + \frac{2(-1)^n(2^{2n}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$

1.27.8 $\int \operatorname{sh}^2 ax\,dx = \frac{\operatorname{sh} ax \operatorname{ch} ax}{2a} - \frac{x}{2}$

1.27.9 $\int x \operatorname{sh}^2 ax\,dx = \frac{x \operatorname{sh} 2ax}{4a} - \frac{\operatorname{ch} 2ax}{8a^2} - \frac{x^2}{4}$

1.27.10 $\int \frac{dx}{\operatorname{sh}^2 ax} = -\frac{\coth ax}{a}$

1.27.11 $\int \operatorname{sh} ax \operatorname{sh} px\,dx = \frac{\operatorname{sh}(a+p)x}{2(a+p)} - \frac{\operatorname{sh}(a-p)x}{2(a-p)}$

Para $a = \pm p$, véase 1.27.8.

1.27.12 $\int x^m \operatorname{sh} ax\,dx = \frac{x^m \operatorname{ch} ax}{a} - \frac{m}{a}\int x^{m-1} \operatorname{ch} ax\,dx$ [Véase 1.28.12].

1.27.13 $\int \operatorname{sh}^n ax\,dx = \frac{\operatorname{sh}^{n-1} ax \operatorname{ch} ax}{an} - \frac{n-1}{n}\int \operatorname{sh}^{n-2} ax\,dx$

1.27.14 $\int \frac{\operatorname{sh} ax}{x^n}dx = \frac{-\operatorname{sh} ax}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{\operatorname{ch} ax}{x^{n-1}}dx$ [Véase 1.28.14].

1.27.15 $\int \frac{dx}{\operatorname{sh}^n ax} = \frac{-\operatorname{ch} ax}{a(n-1)\operatorname{sh}^{n-1} ax} - \frac{n-2}{n-1}\int \frac{dx}{\operatorname{sh}^{n-2} ax}$

1.27.16 $\int \frac{x\,dx}{\operatorname{sh}^n ax} = \frac{-x \operatorname{ch} ax}{a(n-1)\operatorname{sh}^{n-1} ax} - \frac{1}{a^2(n-1)(n-2)\operatorname{sh}^{n-2} ax} - \frac{n-2}{n-1}\int \frac{x\,dx}{\operatorname{sh}^{n-2} ax}$

(28) INTEGRALES QUE CONTIENEN ch ax

1.28.1 $\int \operatorname{ch} ax\,dx = \frac{\operatorname{sh} ax}{a}$

1.28.2 $\int x \operatorname{ch} ax\,dx = \frac{x \operatorname{sh} ax}{a} - \frac{\operatorname{ch} ax}{a^2}$

1.28.3 $\int x^2 \operatorname{ch} ax\,dx = -\frac{2x \operatorname{ch} ax}{a^2} + \left(\frac{x^2}{a} + \frac{2}{a^3}\right)\operatorname{sh} ax$

1.28.4 $\int \frac{\operatorname{ch} ax}{x}dx = \ln x + \frac{(ax)^2}{2\cdot 2!} + \frac{(ax)^4}{4\cdot 4!} + \frac{(ax)^6}{6\cdot 6!} + \cdots$

1.28.5 $\int \frac{\operatorname{ch} ax}{x^2}dx = -\frac{\operatorname{ch} ax}{x} + a\int \frac{\operatorname{sh} ax}{x}dx$ [Véase 1.27.4].

1.28.6 $\int \frac{dx}{\operatorname{ch} ax} = \frac{2}{a}\operatorname{tg}^{-1} e^{ax}$

1.28.7 $$\int \frac{x\,dx}{\operatorname{ch} ax} = \frac{1}{a^2}\left\{\frac{(ax)^2}{2} - \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \cdots + \frac{(-1)^n E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \cdots\right\}$$

1.28.8 $$\int \operatorname{ch}^2 ax\,dx = \frac{x}{2} + \frac{\operatorname{sh} ax \operatorname{ch} ax}{2a}$$

1.28.9 $$\int x \operatorname{ch}^2 ax\,dx = \frac{x^2}{4} + \frac{x \operatorname{sh} 2ax}{4a} - \frac{\operatorname{ch} 2ax}{8a^2}$$

1.28.10 $$\int \frac{dx}{\operatorname{ch}^2 ax} = \frac{\operatorname{th} ax}{a}$$

1.28.11 $$\int \operatorname{ch} ax \operatorname{ch} px\,dx = \frac{\operatorname{sh}(a-p)x}{2(a-p)} + \frac{\operatorname{sh}(a+p)x}{2(a+p)}$$

1.28.12 $$\int x^m \operatorname{ch} ax\,dx = \frac{x^m \operatorname{sh} ax}{a} - \frac{m}{a}\int x^{m-1} \operatorname{sh} ax\,dx$$ [Véase 1.27.12].

1.28.13 $$\int \operatorname{ch}^n ax\,dx = \frac{\operatorname{ch}^{n-1} ax \operatorname{sh} ax}{an} + \frac{n-1}{n}\int \operatorname{ch}^{n-2} ax\,dx$$

1.28.14 $$\int \frac{\operatorname{ch} ax}{x^n}\,dx = \frac{-\operatorname{ch} ax}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{\operatorname{sh} ax}{x^{n-1}}\,dx$$ [Véase 1.27.14].

1.28.15 $$\int \frac{dx}{\operatorname{ch}^n ax} = \frac{\operatorname{sh} ax}{a(n-1)\operatorname{ch}^{n-1} ax} + \frac{n-2}{n-1}\int \frac{dx}{\operatorname{ch}^{n-2} ax}$$

1.28.16 $$\int \frac{x\,dx}{\operatorname{ch}^n ax} = \frac{x \operatorname{sh} ax}{a(n-1)\operatorname{ch}^{n-1} ax} + \frac{1}{(n-1)(n-2)a^2 \operatorname{ch}^{n-2} ax} + \frac{n-2}{n-1}\int \frac{x\,dx}{\operatorname{ch}^{n-2} ax}$$

(29) INTEGRALES QUE CONTIENEN sh ax Y ch ax

1.29.1 $$\int \operatorname{sh} ax \operatorname{ch} ax\,dx = \frac{\operatorname{sh}^2 ax}{2a}$$

1.29.2 $$\int \operatorname{sh} px \operatorname{ch} qx\,dx = \frac{\operatorname{ch}(p+q)x}{2(p+q)} + \frac{\operatorname{ch}(p-q)x}{2(p-q)}$$

1.29.3 $$\int \operatorname{sh}^2 ax \operatorname{ch}^2 ax\,dx = \frac{\operatorname{sh} 4ax}{32a} - \frac{x}{8}$$

1.29.4 $$\int \frac{dx}{\operatorname{sh} ax \operatorname{ch} ax} = \frac{1}{a}\ln \operatorname{th} ax$$

1.29.5 $$\int \frac{dx}{\operatorname{sh}^2 ax \operatorname{ch}^2 ax} = -\frac{2\coth 2ax}{a}$$

1.29.6 $$\int \frac{\operatorname{sh}^2 ax}{\operatorname{ch} ax}\,dx = \frac{\operatorname{sh} ax}{a} - \frac{1}{a}\operatorname{tg}^{-1} \operatorname{sh} ax$$

1.29.7 $\int \frac{\operatorname{ch}^2 ax}{\operatorname{sh} ax}\,dx = \frac{\operatorname{ch} ax}{a} + \frac{1}{a}\ln \operatorname{th}\frac{ax}{2}$

(30) INTEGRALES QUE CONTIENEN th *ax*

1.30.1 $\int \operatorname{th} ax\,dx = \frac{1}{a}\ln \operatorname{ch} ax$

1.30.2 $\int \operatorname{th}^2 ax\,dx = x - \frac{\operatorname{th} ax}{a}$

1.30.3 $\int \operatorname{th}^3 ax\,dx = \frac{1}{a}\ln \operatorname{ch} ax - \frac{\operatorname{th}^2 ax}{2a}$

1.30.4 $\int x \operatorname{th} ax\,dx = \frac{1}{a^2}\left\{\frac{(ax)^3}{3} - \frac{(ax)^5}{15} + \frac{2(ax)^7}{105} - \cdots \frac{(-1)^{n-1}2^{2n}(2^{2n}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$

1.30.5 $\int x \operatorname{th}^2 ax\,dx = \frac{x^2}{2} - \frac{x \operatorname{th} ax}{a} + \frac{1}{a^2}\ln \operatorname{ch} ax$

1.30.6 $\int \frac{\operatorname{th} ax}{x}\,dx = ax - \frac{(ax)^3}{9} + \frac{2(ax)^5}{75} - \cdots \frac{(-1)^{n-1}2^{2n}(2^{2n}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$

1.30.7 $\int \frac{dx}{p + q \operatorname{th} ax} = \frac{px}{p^2 - q^2} - \frac{q}{a(p^2 - q^2)}\ln(q \operatorname{sh} ax + p \operatorname{ch} ax)$

1.30.8 $\int \operatorname{th}^n ax\,dx = \frac{-\operatorname{th}^{n-1} ax}{a(n-1)} + \int \operatorname{th}^{n-2} ax\,dx$

(31) INTEGRALES QUE CONTIENEN coth *ax*

1.31.1 $\int \coth ax\,dx = \frac{1}{a}\ln \operatorname{sh} ax$

1.31.2 $\int \coth^2 ax\,dx = x - \frac{\coth ax}{a}$

1.31.3 $\int \coth^3 ax\,dx = \frac{1}{a}\ln \operatorname{sh} ax - \frac{\coth^2 ax}{2a}$

1.31.4 $\int x \coth ax\,dx = \frac{1}{a^2}\left\{ax + \frac{(ax)^3}{9} - \frac{(ax)^5}{225} + \cdots \frac{(-1)^{n-1}2^{2n}B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$

1.31.5 $\int x \coth^2 ax\,dx = \frac{x^2}{2} - \frac{x \coth ax}{a} + \frac{1}{a^2}\ln \operatorname{sh} ax$

1.31.6 $\displaystyle\int \frac{\coth ax}{x}\,dx = -\frac{1}{ax} + \frac{ax}{3} - \frac{(ax)^3}{135} + \cdots \frac{(-1)^n 2^{2n} B_n (ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$

1.31.7 $\displaystyle\int \frac{dx}{p + q\coth ax} = \frac{px}{p^2 - q^2} - \frac{q}{a(p^2 - q^2)} \ln(p \operatorname{sh} ax + q \operatorname{ch} ax)$

1.31.8 $\displaystyle\int \coth^n ax\,dx = -\frac{\coth^{n-1} ax}{a(n-1)} + \int \coth^{n-2} ax\,dx$

(32) INTEGRALES QUE CONTIENEN sech ax

1.32.1 $\displaystyle\int \operatorname{sech} ax\,dx = \frac{2}{a} \operatorname{tg}^{-1} e^{ax}$

1.32.2 $\displaystyle\int \operatorname{sech}^2 ax\,dx = \frac{\operatorname{th} ax}{a}$

1.32.3 $\displaystyle\int \operatorname{sech}^3 ax\,dx = \frac{\operatorname{sech} ax \operatorname{th} ax}{2a} + \frac{1}{2a} \operatorname{tg}^{-1} \operatorname{sh} ax$

1.32.4 $\displaystyle\int x \operatorname{sech} ax\,dx = \frac{1}{a^2}\left\{\frac{(ax)^2}{2} - \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \cdots \frac{(-1)^n E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \cdots\right\}$

1.32.5 $\displaystyle\int x \operatorname{sech}^2 ax\,dx = \frac{x \operatorname{th} ax}{a} - \frac{1}{a^2} \ln \operatorname{ch} ax$

1.32.6 $\displaystyle\int \frac{\operatorname{sech} ax}{x}\,dx = \ln x - \frac{(ax)^2}{4} + \frac{5(ax)^4}{96} - \frac{61(ax)^6}{4320} + \cdots \frac{(-1)^n E_n (ax)^{2n}}{2n(2n)!} + \cdots$

1.32.7 $\displaystyle\int \operatorname{sech}^n ax\,dx = \frac{\operatorname{sech}^{n-2} ax \operatorname{th} ax}{a(n-1)} + \frac{n-2}{n-1} \int \operatorname{sech}^{n-2} ax\,dx$

(33) INTEGRALES QUE CONTIENEN csch ax

1.33.1 $\displaystyle\int \operatorname{csch} ax\,dx = \frac{1}{a} \ln \operatorname{th} \frac{ax}{2}$

1.33.2 $\displaystyle\int \operatorname{csch}^2 ax\,dx = -\frac{\coth ax}{a}$

1.33.3 $\displaystyle\int \operatorname{csch}^3 ax\,dx = -\frac{\operatorname{csch} ax \coth ax}{2a} - \frac{1}{2a} \ln \operatorname{th} \frac{ax}{2}$

1.33.4 $\displaystyle\int x \operatorname{csch} ax\,dx = \frac{1}{a^2}\left\{ax - \frac{(ax)^3}{18} + \frac{7(ax)^5}{1800} + \cdots + \frac{2(-1)^n (2^{2n-1} - 1) B_n (ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$

1.33.5 $\int x \operatorname{csch}^2 ax\,dx = -\dfrac{x \coth ax}{a} + \dfrac{1}{a^2} \ln \operatorname{sh} ax$

1.33.6 $\int \dfrac{\operatorname{csch} ax}{x}\,dx = -\dfrac{1}{ax} - \dfrac{ax}{6} + \dfrac{7(ax)^3}{1080} + \cdots \dfrac{(-1)^n 2(2^{2n-1}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$

1.33.7 $\int \operatorname{csch}^n ax\,dx = \dfrac{-\operatorname{csch}^{n-2} ax \coth ax}{a(n-1)} - \dfrac{n-2}{n-1} \int \operatorname{csch}^{n-2} ax\,dx$

(34) INTEGRALES QUE CONTIENEN FUNCIONES HIPERBÓLICAS INVERSAS

1.34.1 $\int \operatorname{sh}^{-1} \dfrac{x}{a}\,dx = x \operatorname{sh}^{-1} \dfrac{x}{a} - \sqrt{x^2 + a^2}$

1.34.2 $\int x \operatorname{sh}^{-1} \dfrac{x}{a}\,dx = \left(\dfrac{x^2}{2} + \dfrac{a^2}{4}\right) \operatorname{sh}^{-1} \dfrac{x}{a} - \dfrac{x\sqrt{x^2 + a^2}}{4}$

1.34.3
$$\int \frac{\operatorname{sh}^{-1}(x/a)}{x}\,dx = \begin{cases} \dfrac{x}{a} - \dfrac{(x/a)^3}{2\cdot 3\cdot 3} + \dfrac{1\cdot 3(x/a)^5}{2\cdot 4\cdot 5\cdot 5} - \dfrac{1\cdot 3\cdot 5(x/a)^7}{2\cdot 4\cdot 6\cdot 7\cdot 7} + \cdots & |x| < a \\ \dfrac{\ln^2(2x/a)}{2} - \dfrac{(a/x)^2}{2\cdot 2\cdot 2} + \dfrac{1\cdot 3(a/x)^4}{2\cdot 4\cdot 4\cdot 4} - \dfrac{1\cdot 3\cdot 5(a/x)^6}{2\cdot 4\cdot 6\cdot 6\cdot 6} + \cdots & x > a \\ -\dfrac{\ln^2(-2x/a)}{2} + \dfrac{(a/x)^2}{2\cdot 2\cdot 2} - \dfrac{1\cdot 3(a/x)^4}{2\cdot 4\cdot 4\cdot 4} + \dfrac{1\cdot 3\cdot 5(a/x)^6}{2\cdot 4\cdot 6\cdot 6\cdot 6} - \cdots & x < -a \end{cases}$$

1.34.4
$$\int \operatorname{ch}^{-1} \frac{x}{a}\,dx = \begin{cases} x \operatorname{ch}^{-1}(x/a) - \sqrt{x^2 - a^2}, & \operatorname{ch}^{-1}(x/a) > 0 \\ x \operatorname{ch}^{-1}(x/a) + \sqrt{x^2 - a^2}, & \operatorname{ch}^{-1}(x/a) < 0 \end{cases}$$

1.34.5
$$\int x \operatorname{ch}^{-1} \frac{x}{a}\,dx = \begin{cases} \frac{1}{4}(2x^2 - a^2) \operatorname{ch}^{-1}(x/a) - \frac{1}{4} x\sqrt{x^2 - a^2}, & \operatorname{ch}^{-1}(x/a) > 0 \\ \frac{1}{4}(2x^2 - a^2) \operatorname{ch}^{-1}(x/a) + \frac{1}{4} x\sqrt{x^2 - a^2}, & \operatorname{ch}^{-1}(x/a) < 0 \end{cases}$$

1.34.6
$$\int \frac{\operatorname{ch}^{-1}(x/a)}{x}\,dx = \pm\left[\frac{1}{2}\ln^2(2x/a) + \frac{(a/x)^2}{2\cdot 2\cdot 2} + \frac{1\cdot 3(a/x)^4}{2\cdot 4\cdot 4\cdot 4} + \frac{1\cdot 3\cdot 5(a/x)^6}{2\cdot 4\cdot 6\cdot 6\cdot 6} + \cdots\right]$$

$+$ si $\operatorname{ch}^{-1}(x/a) > 0$, $\quad -$ si $\operatorname{ch}^{-1}(x/a) < 0$.

1.34.7 $\int \operatorname{th}^{-1} \dfrac{x}{a}\,dx = x \operatorname{th}^{-1} \dfrac{x}{a} + \dfrac{a}{2} \ln(a^2 - x^2)$

1.34.8 $\int x \operatorname{th}^{-1} \dfrac{x}{a}\,dx = \dfrac{ax}{2} + \frac{1}{2}(x^2 - a^2) \operatorname{th}^{-1} \dfrac{x}{a}$

1.34.9 $\int \dfrac{\operatorname{th}^{-1}(x/a)}{x}\,dx = \dfrac{x}{a} + \dfrac{(x/a)^3}{3^2} + \dfrac{(x/a)^5}{5^2} + \cdots$

1.34.10 $\int \coth^{-1} \dfrac{x}{a}\,dx = x \coth^{-1} x + \dfrac{a}{2} \ln(x^2 - a^2)$

1.34.11 $$\int x \coth^{-1}\frac{x}{a}\,dx = \frac{ax}{2} + \tfrac{1}{2}(x^2 - a^2)\coth^{-1}\frac{x}{a}$$

1.34.12 $$\int \frac{\coth^{-1}(x/a)}{x}\,dx = -\left(\frac{a}{x} + \frac{(a/x)^3}{3^2} + \frac{(a/x)^5}{5^2} + \cdots\right)$$

1.34.13 $$\int \operatorname{sech}^{-1}\frac{x}{a}\,dx = \begin{cases} x\operatorname{sech}^{-1}(x/a) + a\operatorname{sen}^{-1}(x/a), & \operatorname{sech}^{-1}(x/a) > 0 \\ x\operatorname{sech}^{-1}(x/a) - a\operatorname{sen}^{-1}(x/a), & \operatorname{sech}^{-1}(x/a) < 0 \end{cases}$$

1.34.14 $$\int \operatorname{csch}^{-1}\frac{x}{a}\,dx = x\operatorname{csch}^{-1}\frac{x}{a} \pm a\operatorname{sh}^{-1}\frac{x}{a} \qquad [+\text{ si } x > 0, \quad -\text{ si } x < 0].$$

1.34.15 $$\int x^m \operatorname{sh}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{sh}^{-1}\frac{x}{a} - \frac{1}{m+1}\int \frac{x^{m+1}}{\sqrt{x^2 + a^2}}\,dx$$

1.34.16 $$\int x^m \operatorname{ch}^{-1}\frac{x}{a}\,dx = \begin{cases} \dfrac{x^{m+1}}{m+1}\operatorname{ch}^{-1}\dfrac{x}{a} - \dfrac{1}{m+1}\displaystyle\int \frac{x^{m+1}}{\sqrt{x^2 - a^2}}\,dx & \operatorname{ch}^{-1}(x/a) > 0 \\ \dfrac{x^{m+1}}{m+1}\operatorname{ch}^{-1}\dfrac{x}{a} + \dfrac{1}{m+1}\displaystyle\int \frac{x^{m+1}}{\sqrt{x^2 - a^2}}\,dx & \operatorname{ch}^{-1}(x/a) < 0 \end{cases}$$

1.34.17 $$\int x^m \operatorname{th}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{th}^{-1}\frac{x}{a} - \frac{a}{m+1}\int \frac{x^{m+1}}{a^2 - x^2}\,dx$$

1.34.18 $$\int x^m \coth^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\coth^{-1}\frac{x}{a} - \frac{1}{m+1}\int \frac{x^{m+1}}{a^2 - x^2}\,dx$$

1.34.19 $$\int x^m \operatorname{sech}^{-1}\frac{x}{a}\,dx = \begin{cases} \dfrac{x^{m+1}}{m+1}\operatorname{sech}^{-1}\dfrac{x}{a} + \dfrac{a}{m+1}\displaystyle\int \frac{x^m\,dx}{\sqrt{a^2 - x^2}} & \operatorname{sech}^{-1}(x/a) > 0 \\ \dfrac{x^{m+1}}{m+1}\operatorname{sech}^{-1}\dfrac{x}{a} - \dfrac{a}{m+1}\displaystyle\int \frac{x^m\,dx}{\sqrt{a^2 - x^2}} & \operatorname{sech}^{-1}(x/a) < 0 \end{cases}$$

1.34.20 $$\int x^m \operatorname{csch}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{csch}^{-1}\frac{x}{a} \pm \frac{a}{m+1}\int \frac{x^m\,dx}{\sqrt{x^2 + a^2}} \qquad [+\text{ si } x > 0, \quad -\text{ si } x < 0].$$

TABLA

2 TABLA DE INTEGRALES DEFINIDAS

INTEGRALES DEFINIDAS QUE CONTIENEN EXPRESIONES RACIONALES O IRRACIONALES

2.1 $$\int_0^\infty \frac{dx}{x^2 + a^2} = \frac{\pi}{2a}$$

2.2 $$\int_0^\infty \frac{x^{p-1}\,dx}{1 + x} = \frac{\pi}{\operatorname{sen} p\pi}, \qquad 0 < p < 1$$

2.3 $$\int_0^\infty \frac{x^m\,dx}{x^n + a^n} = \frac{\pi a^{m+1-n}}{n \operatorname{sen}[(m+1)\pi/n]}, \qquad 0 < m + 1 < n$$

2.4 $$\int_0^\infty \frac{x^m\,dx}{1 + 2x\cos\beta + x^2} = \frac{\pi}{\operatorname{sen} m\pi}\frac{\operatorname{sen} m\beta}{\operatorname{sen}\beta}$$

2.5 $$\int_0^a \frac{dx}{\sqrt{a^2 - x^2}} = \frac{\pi}{2}$$

2.6 $$\int_0^a \sqrt{a^2 - x^2}\,dx = \frac{\pi a^2}{4}$$

2.7 $$\int_0^a x^m(a^n - x^n)^p\,dx = \frac{a^{m+1+np}\Gamma[(m+1)/n]\Gamma(p+1)]}{n\Gamma(m+1)/n + p + 1]}$$

2.8 $$\int_0^\infty \frac{x^m\,dx}{(x^n + a^n)^r} = \frac{(-1)^{r-1}\pi a^{m+1-nr}\Gamma[(m+1)/n]}{n \operatorname{sen}[(m+1)\pi/n](r-1)!\Gamma[(m+1)/n - r + 1]}, \qquad 0 < m + 1 < nr$$

INTEGRALES DEFINIDAS QUE CONTIENEN FUNCIONES TRIGONOMÉTRICAS

Todos los parámetros se consideran positivos, a no ser que se indique lo contrario.

2.9 $$\int_0^\pi \operatorname{sen} mx \operatorname{sen} nx\,dx = \begin{cases} 0 & m,\ n \text{ enteros y } m \neq n \\ \pi/2 & m,\ n \text{ enteros y } m = n \end{cases}$$

2.10 $$\int_0^\pi \cos mx \cos nx\,dx = \begin{cases} 0 & m,\ n \text{ enteros y } m \neq n \\ \pi/2 & m,\ n \text{ enteros y } m = n \end{cases}$$

2.11 $$\int_0^\pi \operatorname{sen} mx \cos nx\,dx = \begin{cases} 0 & m,\ n \text{ enteros y } m + n \text{ impar} \\ 2m/(m^2 - n^2) & m,\ n \text{ enteros y } m + n \text{ par} \end{cases}$$

2.12 $$\int_0^{\pi/2} \operatorname{sen}^2 x\,dx = \int_0^{\pi/2} \cos^2 x\,dx = \frac{\pi}{4}$$

2.13 $\displaystyle\int_0^{\pi/2} \operatorname{sen}^{2m} x\,dx = \int_0^{\pi/2} \cos^{2m} x\,dx = \frac{1\cdot 3\cdot 5\cdots 2m-1}{2\cdot 4\cdot 6\cdots 2m}\,\frac{\pi}{2}, \qquad m = 1, 2, \ldots$

2.14 $\displaystyle\int_0^{\pi/2} \operatorname{sen}^{2m+1} x\,dx = \int_0^{\pi/2} \cos^{2m+1} x\,dx = \frac{2\cdot 4\cdot 6\cdots 2m}{1\cdot 3\cdot 5\cdots 2m+1}, \qquad m = 1, 2, \ldots$

2.15 $\displaystyle\int_0^{\pi/2} \operatorname{sen}^{2p-1} x \cos^{2q-1} x\,dx = \frac{\Gamma(p)\Gamma(q)}{2\Gamma(p+q)}$

2.16 $\displaystyle\int_0^{\infty} \frac{\operatorname{sen} px}{x}\,dx = \begin{cases} \pi/2 & p > 0 \\ 0 & p = 0 \\ -\pi/2 & p < 0 \end{cases}$

2.17 $\displaystyle\int_0^{\infty} \frac{\operatorname{sen} px \cos qx}{x}\,dx = \begin{cases} 0 & p > q > 0 \\ \pi/2 & 0 < p < q \\ \pi/4 & p = q > 0 \end{cases}$

2.18 $\displaystyle\int_0^{\infty} \frac{\operatorname{sen} px \operatorname{sen} qx}{x^2}\,dx = \begin{cases} \pi p/2 & 0 < p \leqslant q \\ \pi p/2 & p \geqslant q > 0 \end{cases}$

2.19 $\displaystyle\int_0^{\infty} \frac{\operatorname{sen}^2 px}{x^2}\,dx = \frac{\pi p}{2}$

2.20 $\displaystyle\int_0^{\infty} \frac{1 - \cos px}{x^2}\,dx = \frac{\pi p}{2}$

2.21 $\displaystyle\int_0^{\infty} \frac{\cos px - \cos qx}{x}\,dx = \ln \frac{q}{p}$

2.22 $\displaystyle\int_0^{\infty} \frac{\cos px - \cos qx}{x^2}\,dx = \frac{\pi(q-p)}{2}$

2.23 $\displaystyle\int_0^{\infty} \frac{\cos mx}{x^2 + a^2}\,dx = \frac{\pi}{2a}\,e^{-ma}$

2.24 $\displaystyle\int_0^{\infty} \frac{x \operatorname{sen} mx}{x^2 + a^2}\,dx = \frac{\pi}{2}\,e^{-ma}$

2.25 $\displaystyle\int_0^{\infty} \frac{\operatorname{sen} mx}{x(x^2 + a^2)}\,dx = \frac{\pi}{2a^2}\,(1 - e^{-ma})$

2.26 $\displaystyle\int_0^{2\pi} \frac{dx}{a + b \operatorname{sen} x} = \frac{2\pi}{\sqrt{a^2 - b^2}}$

2.27 $\displaystyle\int_0^{2\pi} \frac{dx}{a + b \cos x} = \frac{2\pi}{\sqrt{a^2 - b^2}}$

2.28 $\displaystyle\int_0^{\pi/2} \frac{dx}{a + b \cos x} = \frac{\cos^{-1}(b/a)}{\sqrt{a^2 - b^2}}$

2.29 $\displaystyle\int_0^{2\pi} \frac{dx}{(a + b \operatorname{sen} x)^2} = \int_0^{2\pi} \frac{dx}{(a + b \cos x)^2} = \frac{2\pi a}{(a^2 - b^2)^{3/2}}$

2.30 $\displaystyle\int_0^{2\pi} \frac{dx}{1 - 2a \cos x + a^2} = \frac{2\pi}{1 - a^2}, \qquad 0 < a < 1$

2.31 $\displaystyle\int_0^{\pi} \frac{x \operatorname{sen} x\,dx}{1 - 2a \cos x + a^2} = \begin{cases} (\pi/a) \ln(1 + a), & |a| < 1 \\ \pi \ln(1 + 1/a), & |a| > 1 \end{cases}$

2.32 $\displaystyle\int_0^{\pi} \frac{\cos mx\,dx}{1 - 2a \cos x + a^2} = \frac{\pi a^m}{1 - a^2}, \quad a^2 < 1, \quad m = 0, 1, 2, \ldots$

2.33 $\displaystyle\int_0^{\infty} \operatorname{sen} ax^2\,dx = \int_0^{\infty} \cos ax^2\,dx = \frac{1}{2}\sqrt{\frac{\pi}{2a}}$

2.34 $$\int_0^\infty \operatorname{sen} ax^n\,dx = \frac{1}{na^{1/n}}\,\Gamma(1/n)\operatorname{sen}\frac{\pi}{2n}, \qquad n > 1$$

2.35 $$\int_0^\infty \cos ax^n\,dx = \frac{1}{na^{1/n}}\,\Gamma(1/n)\cos\frac{\pi}{2n}, \qquad n > 1$$

2.36 $$\int_0^\infty \frac{\operatorname{sen} x}{\sqrt{x}}\,dx = \int_0^\infty \frac{\cos x}{\sqrt{x}}\,dx = \sqrt{\frac{\pi}{2}}$$

2.37 $$\int_0^\infty \frac{\operatorname{sen} x}{x^p}\,dx = \frac{\pi}{2\Gamma(p)\operatorname{sen}(p\pi/2)}, \qquad 0 < p < 1$$

2.38 $$\int_0^\infty \frac{\cos x}{x^p}\,dx = \frac{\pi}{2\Gamma(p)\cos(p\pi/2)}, \qquad 0 < p < 1$$

2.39 $$\int_0^\infty \operatorname{sen} ax^2 \cos 2bx\,dx = \frac{1}{2}\sqrt{\frac{\pi}{2a}}\left(\cos\frac{b^2}{a} - \operatorname{sen}\frac{b^2}{a}\right)$$

2.40 $$\int_0^\infty \cos ax^2 \cos 2bx\,dx = \frac{1}{2}\sqrt{\frac{\pi}{2a}}\left(\cos\frac{b^2}{a} + \operatorname{sen}\frac{b^2}{a}\right)$$

2.41 $$\int_0^\infty \frac{\operatorname{sen}^3 x}{x^3}\,dx = \frac{3\pi}{8}$$

2.42 $$\int_0^\infty \frac{\operatorname{sen}^4 x}{x^4}\,dx = \frac{\pi}{3}$$

2.43 $$\int_0^\infty \frac{\operatorname{tg} x}{x}\,dx = \frac{\pi}{2}$$

2.44 $$\int_0^{\pi/2} \frac{dx}{1 + \operatorname{tg}^m x} = \frac{\pi}{4}$$

2.45 $$\int_0^{\pi/2} \frac{x}{\operatorname{sen} x}\,dx = 2\left\{\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots\right\}$$

2.46 $$\int_0^1 \frac{\operatorname{tg}^{-1} x}{x}\,dx = \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots$$

2.47 $$\int_0^1 \frac{\operatorname{sen}^{-1} x}{x}\,dx = \frac{\pi}{2}\ln 2$$

2.48 $$\int_0^1 \frac{1 - \cos x}{x}\,dx - \int_1^\infty \frac{\cos x}{x}\,dx = \gamma$$ (γ es la constante de Euler).

2.49 $$\int_0^\infty \left(\frac{1}{1 + x^2} - \cos x\right)\frac{dx}{x} = \gamma$$

2.50 $$\int_0^\infty \frac{\operatorname{tg}^{-1} px - \operatorname{tg}^{-1} qx}{x}\,dx = \frac{\pi}{2}\ln\frac{p}{q}$$

INTEGRALES DEFINIDAS QUE CONTIENEN FUNCIONES EXPONENCIALES

2.51 $$\int_0^\infty e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2}$$

2.52 $$\int_0^\infty e^{-ax}\operatorname{sen} bx\,dx = \frac{b}{a^2+b^2}$$

2.53 $$\int_0^\infty \frac{e^{-ax}\operatorname{sen} bx}{x}\,dx = \operatorname{tg}^{-1}\frac{b}{a}$$

2.54 $$\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}\,dx = \ln\frac{b}{a}$$

2.55 $$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

2.56 $$\int_0^\infty e^{-ax^2}\cos bx\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,e^{-b^2/4a}$$

2.57 $$\int_0^\infty e^{-(ax^2+bx+c)}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,e^{(b^2-4ac)/4a}\operatorname{fcer}\frac{b}{2\sqrt{a}}$$

donde $\operatorname{fcer}(p) = \dfrac{2}{\sqrt{\pi}}\displaystyle\int_p^\infty e^{-x^2}\,dx$

2.58 $$\int_0^\infty e^{-(ax^2+bx+c)}\,dx = \sqrt{\frac{\pi}{a}}\,e^{(b^2-4ac)/4a}$$

2.59 $$\int_0^\infty x^n e^{-ax}\,dx = \frac{\Gamma(n+1)}{a^{n+1}}$$

2.60 $$\int_0^\infty x^m e^{-ax^2}\,dx = \frac{\Gamma[(m+1)/2]}{2a^{(m+1)/2}}$$

2.61 $$\int_0^\infty e^{-(ax^2+b/x^2)}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,e^{-2\sqrt{ab}}$$

2.62 $$\int_0^\infty \frac{x\,dx}{e^x-1} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots = \frac{\pi^2}{6}$$

2.63 $$\int_0^\infty \frac{x^{n-1}}{e^x-1}\,dx = \Gamma(n+1)\left(\frac{1}{1^n}+\frac{1}{2^n}+\frac{1}{3^n}+\cdots\right)$$

Cuando n es par, esta última suma se puede hallar con ayuda de los números de Bernoulli [véanse páginas 135-136].

2.64 $$\int_0^\infty \frac{x\,dx}{e^x+1} = \frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots = \frac{\pi^2}{12}$$

2.65 $$\int_0^\infty \frac{x^{n-1}}{e^x+1}\,dx = \Gamma(n+1)\left(\frac{1}{1^n} - \frac{1}{2^n} + \frac{1}{3^n} - \cdots\right)$$

La suma de esta última serie puede encontrarse para ciertos valores enteros positivos de n [véanse páginas 127-128 y 135-136].

2.66 $$\int_0^\infty \frac{\operatorname{sen} mx}{e^{2\pi x}-1}\,dx = \frac{1}{4}\coth\frac{m}{2} - \frac{1}{2m}$$

2.67 $$\int_0^\infty \left(\frac{1}{1+x} - e^{-x}\right)\frac{dx}{x} = \gamma$$ (γ es la constante de Euler).

2.68 $$\int_0^\infty \frac{e^{-x^2} - e^{-x}}{x}\,dx = \tfrac{1}{2}\gamma$$

2.69 $$\int_0^\infty \left(\frac{1}{e^x-1} - \frac{e^{-x}}{x}\right)dx = \gamma$$

2.70 $$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x \sec px}\,dx = \frac{1}{2}\ln\left(\frac{b^2+p^2}{a^2+p^2}\right)$$

2.71 $$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x \operatorname{cosec} px}\,dx = \operatorname{tg}^{-1}\frac{b}{p} - \operatorname{tg}^{-1}\frac{a}{p}$$

2.72 $$\int_0^\infty \frac{e^{-ax}(1-\cos x)}{x^2}\,dx = \operatorname{cotg}^{-1} a - \frac{a}{2}\ln(a^2+1)$$

INTEGRALES DEFINIDAS QUE CONTIENEN FUNCIONES LOGARÍTMICAS

2.73 $$\int_0^1 x^m (\ln x)^n\,dx = \frac{(-1)^n n!}{(m+1)^{n+1}}, \qquad m > -1,\ n = 0, 1, 2, \ldots$$

Si $n \neq 0, 1, 2, \ldots$, reemplácese $n!$ por $\Gamma(n+1)$.

2.74 $$\int_0^1 \frac{\ln x}{1+x}\,dx = -\frac{\pi^2}{12}$$

2.75 $$\int_0^\infty \frac{\ln x}{1-x}\,dx = -\frac{\pi^2}{6}$$

2.76 $$\int_0^1 \frac{\ln(1+x)}{x}\,dx = \frac{\pi^2}{12}$$

2.77 $$\int_0^1 \frac{\ln(1-x)}{x}\,dx = -\frac{\pi^2}{6}$$

2.78 $$\int_0^1 \ln x \ln(1+x)\,dx = 2 - 2\ln 2 - \frac{\pi^2}{12}$$

2.79 $\displaystyle\int_0^1 \ln x \ln(1-x)\,dx = 2 - \frac{\pi^2}{6}$

2.80 $\displaystyle\int_0^\infty \frac{x^{p-1}\ln x}{1+x}\,dx = -\pi^2 \operatorname{cosec} p\pi \operatorname{cotg} p\pi \qquad 0 < p < 1$

2.81 $\displaystyle\int_0^1 \frac{x^m - x^n}{\ln x}\,dx = \ln\frac{m+1}{n+1}$

2.82 $\displaystyle\int_0^\infty e^{-x}\ln x\,dx = -\gamma$ (γ es la constante de Euler).

2.83 $\displaystyle\int_0^\infty e^{-x^2}\ln x\,dx = -\frac{\sqrt{\pi}}{4}(\gamma + 2\ln 2)$

2.84 $\displaystyle\int_0^\infty \ln\left(\frac{e^x+1}{e^x-1}\right)dx = \frac{\pi^2}{4}$

2.85 $\displaystyle\int_0^{\pi/2} \ln \operatorname{sen} x\,dx = \int_0^{\pi/2} \ln\cos x\,dx = -\frac{\pi}{2}\ln 2$

2.86 $\displaystyle\int_0^{\pi/2} (\ln \operatorname{sen} x)^2\,dx = \int_0^{\pi/2} (\ln\cos x)^2\,dx = \frac{\pi}{2}(\ln 2)^2 + \frac{\pi^3}{24}$

2.87 $\displaystyle\int_0^{\pi} x\ln \operatorname{sen} x\,dx = -\frac{\pi^2}{2}\ln 2$

2.88 $\displaystyle\int_0^{\pi/2} \operatorname{sen} x \ln \operatorname{sen} x\,dx = \ln 2 - 1$

2.89 $\displaystyle\int_0^{2\pi} \ln(a + b\operatorname{sen} x)\,dx = \int_0^{2\pi} \ln(a + b\cos x)\,dx = 2\pi\ln(a + \sqrt{a^2 - b^2})$

2.90 $\displaystyle\int_0^{\pi} \ln(a + b\cos x)\,dx = \pi\ln\left(\frac{a + \sqrt{a^2-b^2}}{2}\right)$

2.91 $\displaystyle\int_0^{\pi} \ln(a^2 - 2ab\cos x + b^2)\,dx = \begin{cases} 2\pi\ln a, & a \geqslant b > 0 \\ 2\pi\ln b, & b \geqslant a > 0 \end{cases}$

2.92 $\displaystyle\int_0^{\pi/4} \ln(1 + \operatorname{tg} x)\,dx = \frac{\pi}{8}\ln 2$

2.93 $\displaystyle\int_0^{\pi/2} \sec x \ln\left(\frac{1 + b\cos x}{1 + a\cos x}\right)dx = \tfrac{1}{2}\{(\cos^{-1} a)^2 - (\cos^{-1} b)^2\}$

2.94 $\displaystyle\int_0^{a} \ln\left(2\operatorname{sen}\frac{x}{2}\right)dx = -\left(\frac{\operatorname{sen} a}{1^2} + \frac{\operatorname{sen} 2a}{2^2} + \frac{\operatorname{sen} 3a}{3^2} + \cdots\right)$

[Véase además 2.85].

INTEGRALES DEFINIDAS QUE CONTIENEN FUNCIONES HIPERBÓLICAS

2.95 $$\int_0^\infty \frac{\operatorname{sen} ax}{\operatorname{sh} bx}\,dx = \frac{\pi}{2b}\operatorname{th}\frac{a\pi}{2b}$$

2.96 $$\int_0^\infty \frac{\cos ax}{\operatorname{ch} bx}\,dx = \frac{\pi}{2b}\operatorname{sech}\frac{a\pi}{2b}$$

2.97 $$\int_0^\infty \frac{x\,dx}{\operatorname{sh} ax} = \frac{\pi^2}{4a^2}$$

2.98 $$\int_0^\infty \frac{x^n\,dx}{\operatorname{sh} ax} = \frac{2^{n+1}-1}{2^n a^{n+1}}\Gamma(n+1)\left\{\frac{1}{1^{n+1}} + \frac{1}{2^{n+1}} + \frac{1}{3^{n+1}} + \cdots\right\}$$

La suma de esta última serie puede hallarse si n es entero positivo e impar [véanse páginas 127-128].

2.99 $$\int_0^\infty \frac{\operatorname{sh} ax}{e^{bx}+1}\,dx = \frac{\pi}{2b}\operatorname{cosec}\frac{a\pi}{b} - \frac{1}{2a}$$

2.100 $$\int_0^\infty \frac{\operatorname{sh} ax}{e^{bx}-1}\,dx = \frac{1}{2a} - \frac{\pi}{2b}\operatorname{cotg}\frac{a\pi}{b}$$

INTEGRALES DEFINIDAS DIVERSAS

2.101 $$\int_0^\infty \frac{f(ax) - f(bx)}{x}\,dx = \{f(0) - f(\infty)\}\ln\frac{b}{a}$$

Esta última se conoce como *integral de Frullani*. Es válida si $f(x)$ es continua y si $\int_1^\infty \frac{f(x) - f(\infty)}{x}\,dx$ converge.

2.102 $$\int_0^1 \frac{dx}{x^x} = \frac{1}{1^1} + \frac{1}{2^2} + \frac{1}{3^3} + \cdots$$

2.103 $$\int_{-a}^{a} (a+x)^{m-1}(a-x)^{n-1}\,dx = (2a)^{m+n-1}\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$