Trabajo Práctico Álgebra de boole.

(1) a) Demostrar, sin usar tabla, la ley de absorción $x(x+y)=x$

$$x(x+y)=x$$
$$x.x+x.y=x \quad \text{(Distributiva)}$$
$$x+xy=x \quad \text{(Idempotencia)}$$
$$x=x \quad \text{(Absorción)}$$

b) Demostrar, sin usar tabla, la ley de DeMorgan $(\overline{xy})=\bar{x}+\bar{y}$

Usando la ley de complemento $x+\bar{x}=1 \wedge x.\bar{x}=0$

$xy+(\overline{xy})=1 \qquad x.\bar{x}=0$

(Resueltos en el video "Álgebra de Boole.")

c) Demostrar usando tabla de valores: $\begin{cases} x+\bar{x}y=x+y \Rightarrow \text{Absorción} \\ x.(\bar{x}+y)=xy \end{cases}$

$\boxed{x+\bar{x}y=x+y}$ → Resuelto en vídeo "Álgebra de Boole"

x	y	$\bar{x}$	$\bar{x}+y$	$x.(\bar{x}+y)$	xy
1	1	0	1	1	1
1	0	0	0	0	0
0	1	1	1	0	0
0	0	1	0	0	0

→ $x(\bar{x}+y)=xy$.

d) $x(y+z)=(xy)+(xz)$

x	y	z	$(y+z)$	$x(y+z)$	(xy)	(xz)	$(xy)+(xz)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

(2) Para qué valores de las variables booleanas x e y se satisface la siguiente igualdad $xy = x+y$?

$$xy = x+y.$$

para $x=0 \wedge y=0$ $\quad 0.0 = 0+0$

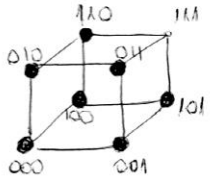
para $x=1 \wedge y=1$ $\quad 1.1 = 1+1$

(3) Calcula los valores y representalos mediante un Q3.

a) $\overline{F}(x,y,z) = x+yz$ → (Resuelto en el video "Funciones y expresiones booleanas")

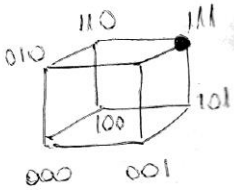
b) $F(x,y,z) = x\bar{y} + (\overline{xyz})$

x	y	z	$\bar{y}$	$x\bar{y}$	xyz	$(\overline{xyz})$	$x\bar{y} + (\overline{xyz})$
1	1	1	0	0	1	0	0
1	1	0	0	0	0	1	1
1	0	1	1	1	0	1	1
1	0	0	1	1	0	1	1
0	1	1	0	0	0	1	1
0	1	0	0	0	0	1	1
0	0	1	1	0	0	1	1
0	0	0	1	0	0	1	1



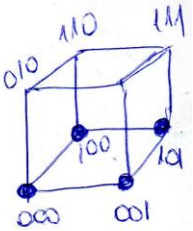
c) $F(x,y,z) = y.(xz + \bar{y}\bar{z})$

x	y	z	xz	$\bar{y}$	$\bar{z}$	$\bar{y}.\bar{z}$	$xz + \bar{y}\bar{z}$	$y(xz+\bar{y}\bar{z})$
1	1	1	1	0	0	0	1	1
1	1	0	0	0	1	0	0	0
1	0	1	1	1	0	0	1	0
1	0	0	0	1	1	1	1	0
0	1	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0
0	0	1	0	1	0	0	0	0
0	0	0	0	1	1	1	1	0



3d) $F(x,y,z) = \bar{y} \cdot (xz + \overline{xz})$

x	y	z	xz	$\overline{xz}$	$(xz + \overline{xz})$	$\bar{y}$	$\bar{y} \cdot (xz + \overline{xz})$
1	1	1	1	0	1	0	0
1	1	0	0	1	1	0	0
1	0	1	1	0	1	1	1
1	0	0	0	1	1	1	1
0	1	1	0	1	1	0	0
0	1	0	0	1	1	0	0
0	0	1	0	1	1	1	1
0	0	0	0	1	1	1	1



4) a)

i) $x \oplus 0$

Si $x=0$ $\quad 0 \oplus 0 = 0$

$x=1$ $\quad 1 \oplus 0 = 1$

ii) $x \oplus 1$

Si $x=0$ $\quad 0 \oplus 1 = 1$

$x=1$ $\quad 1 \oplus 1 = 0$

iii) $x \oplus \bar{x}$

Si $x=0$ $\quad 0 \oplus 1 = 1$

$x=1$ $\quad 1 \oplus 0 = 1$

iv) $x \oplus x$

Si $x=0$ $\quad 0 \oplus 0 = 0$

$x=1$ $\quad 1 \oplus 1 = 0$

b) Demostrar

i) $x \oplus y = (x+y) \cdot (\overline{x \cdot y})$

x	y	(x+y)	xy	$\overline{xy}$	$(x+y) \cdot (\overline{xy})$	$x \oplus y$
1	1	1	1	0	0	0
1	0	1	0	1	1	1
0	1	1	0	1	1	1
0	0	0	0	1	0	0

4b) ii) $x \oplus y = (x\bar{y})(\bar{x}y)$

x	y	$\bar{y}$	$(x\bar{y})$	$\bar{x}$	$(\bar{x}y)$	$(x\bar{y})(\bar{x}y)$	$x \oplus y$
1	1	0	0	0	0	0	0
1	0	1	1	0	0	0	1
0	1	0	0	1	1	0	1
0	0	1	0	1	0	0	0

"no se cumple la igual

iii) $x \oplus y = y \oplus x$

x	y	$x \oplus y$	$y \oplus x$
1	1	0	0
1	0	1	1
0	1	1	1
0	0	0	0

(5) Hallar la forma normal disyuntiva usando tabla y por propiedades.

i) $F(x,y,z) = (x+z).y$

x	y	z	$(x+z)$	$(x+z).y$	
1	1	1	1	1	$\rightarrow xyz$
1	1	0	1	1	$\rightarrow xy\bar{z}$
1	0	1	1	0	
1	0	0	1	0	
0	1	1	1	1	$\rightarrow \bar{x}yz$
0	1	0	0	0	
0	0	1	1	0	
0	0	0	0	0	

$\boxed{F = xyz + xy\bar{z} + \bar{x}yz}$

$F = (x+z).y$

$F = xy + zy$

$F = x.y.1 + 1.zy$

$F = xy(z+\bar{z}) + (x+\bar{x})$

$F = xyz + xy\bar{z} + xyz$

$\boxed{F = xyz + xy\bar{z} + \bar{x}yz}$

5) ii) $F(x,y,z) = \bar{x} + y$

x	y	z	$\bar{x}$	$\bar{x}+y$
1	1	1	0	1 → xyz
1	1	0	0	1 → $xy\bar{z}$
1	0	1	0	0
1	0	0	0	0
0	1	1	1	1 → $\bar{x}yz$
0	1	0	1	1 → $\bar{x}y\bar{z}$
0	0	1	1	1 → $\bar{x}\bar{y}z$
0	0	0	1	1 → $\bar{x}\bar{y}\bar{z}$

$F = xyz + xy\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$

$F = \bar{x} + y$

$F = \bar{x} \cdot 1 \cdot 1 + y \cdot 1 \cdot 1$

$F = \bar{x}(y+\bar{y})(z+\bar{z}) + y(x+\bar{x})(z+\bar{z})$

$F = \underbrace{\bar{x}yz}_{1} + \underbrace{\bar{x}y\bar{z}}_{2} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} + yxz + yx\bar{z} + \underbrace{y\bar{x}z}_{1} + \underbrace{y\bar{x}\bar{z}}_{2}$ Idempotência

$F = \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} + yxz + yx\bar{z}$

iii) $F(x,y,z) = x\bar{y}$

x	y	z	$\bar{y}$	$x\bar{y}$
1	1	1	0	0
1	1	0	0	0
1	0	1	1	1 → $x\bar{y}z$
1	0	0	1	1 → $x\bar{y}\bar{z}$
0	1	1	0	0
0	1	0	0	0
0	0	1	1	0
0	0	0	1	0

$F = x\bar{y}z + x\bar{y}\bar{z}$

$F = x\bar{y}$

$F = x\bar{y} \cdot 1$

$F = x\bar{y}(z+\bar{z})$

$F = x\bar{y}z + x\bar{y}\bar{z}$

5 iv) $F(x,y,z) = (\overline{x} \cdot \overline{z}) \cdot \overline{y}$

x	y	z	$\overline{x}$	$\overline{y}$	$\overline{z}$	$(\overline{x} \cdot \overline{z})$	$(\overline{x} \cdot \overline{z}) \cdot y$
1	1	1	0	0	0	0	0
1	1	0	0	0	1	0	0
1	0	1	0	1	0	0	0
1	0	0	0	1	1	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	1	0	0
0	0	1	1	1	0	0	0
0	0	0	1	1	1	1	1 → $\overline{x}\overline{y}\overline{z}$

$F = \overline{x}\overline{y}\overline{z}$

$F = (\overline{x}\,\overline{z})\,\overline{y}$

$F = \overline{x}\overline{y}\overline{z}$ | conmutativa

(5) v) $F(x,y,z) = \overline{(\overline{x+z}) + (\overline{x} \cdot y)}$

x	y	z	$(x+z)$	$\overline{x+z}$	$\overline{x}$	$\overline{x} \cdot y$	$(\overline{x+z}) + (\overline{x}.y)$	$\overline{(\overline{x+z}) + (\overline{x}y)}$
1	1	1	1	0	0	0	0	1 → xyz
1	1	0	1	0	0	0	0	1 → $xy\overline{z}$
1	0	1	1	0	0	0	0	1 → $x\overline{y}z$
1	0	0	1	0	0	0	0	1 → $x\overline{y}\overline{z}$
0	1	1	1	0	1	1	1	0
0	1	0	0	1	1	1	1	0
0	0	1	1	0	1	0	0	1 → $\overline{x}\overline{y}z$
0	0	0	0	1	1	0	1	0

$F = xyz + xy\overline{z} + x\overline{y}z + x\overline{y}\overline{z} + \overline{x}\overline{y}z$ → F. normal disyuntiva

$\overline{F} = \overline{(\overline{x+z}) + (\overline{x} \cdot y)}$ → Por propiedades.

$F = \overline{\overline{x+z} \cdot \overline{x} \cdot y}$ → De De Morgan

$F = (x+z)(\overline{\overline{x}} + \overline{y})$

$F = (x+z)(x+\overline{y})$

$F = x \cdot x + xy + z \cdot x + z \cdot \overline{y}$

$F = x + xy + zx + z\overline{y}$ Idemp.

$F = x \cdot 1 \cdot 1 + xy \cdot 1 + zx \cdot 1 + 1 \cdot z \cdot \overline{y}$

$F = x(y+\overline{y})(z+\overline{z}) + xy(z+\overline{z}) + zx(y+\overline{y}) + (x+\overline{x})z\overline{y}$

$F = \overbrace{xyz + xy\overline{z}}^{1} + \overbrace{x\overline{y}z + x\overline{y}\overline{z}}^{2} + \overbrace{xyz + \overline{x}yz}^{1} + \overbrace{x\overline{y}z + \overline{x}\overline{y}z}^{2}$

$F = xyz + xy\overline{z} + x\overline{y}z + x\overline{y}\overline{z} + \overline{x}yz + \overline{x}\overline{y}z$

(6) Hallar la forma normal disyuntiva de la función $F(x,y,z)$ que vale 1 si y sólo si:

i) $x \cdot y = 0$ →
- $x=1, y=0$ ← $z=0 \to x\bar{y}\bar{z}$; $z=1 \to x\bar{y}z$
- $x=0, y=1$ ← $z=0 \to \bar{x}y\bar{z}$; $z=1 \to \bar{x}yz$
- $x=0, y=0$ ← $z=0 \to \bar{x}\bar{y}\bar{z}$; $z=1 \to \bar{x}\bar{y}z$.

$$F(x,y,z) = x\bar{y}\bar{z} + x\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z$$

ii) $y=0$

$x=0, y=0, z=0 \to \bar{x}\bar{y}\bar{z}$
$x=0, y=0, z=1 \to \bar{x}\bar{y}z$
$x=1, y=0, z=0 \to x\bar{y}\bar{z}$
$x=1, y=0, z=1 \to x\bar{y}z$

$$F = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + x\bar{y}\bar{z} + x\bar{y}z$$

iii) $x \cdot y \cdot z = 0$

$x=0, y=0, z=0 \to \bar{x}\bar{y}\bar{z}$
$x=0, y=0, z=1 \to \bar{x}\bar{y}z$
$x=0, y=1, z=0 \to \bar{x}y\bar{z}$
$x=0, y=1, z=1 \to \bar{x}yz$
$x=1, y=0, z=0 \to x\bar{y}\bar{z}$
$x=1, y=0, z=1 \to x\bar{y}z$
$x=1, y=1, z=0 \to xy\bar{z}$
$x=1, y=1, z=1 \to xyz$

$$F = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}\bar{z} + x\bar{y}z + xy\bar{z} + xyz$$

(1) Simplificar las expresiones booleanas por propiedades.

i) $F = wxyz + wx\bar{y}z + wx\bar{y}\bar{z} + w\bar{x}y\bar{z} + w\bar{x}\bar{y}z$

(Resuelto en video "Simplificación de funciones booleanas")

ii) $F = wxy\bar{z} + wx\bar{y}z + w\bar{x}yz + \bar{w}x\bar{y}z + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}\bar{y}z$

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$	
wx		1		(1)	→ 3
$w\bar{x}$	1				
$\bar{w}\bar{x}$		1		(1)	→ 1
$\bar{w}x$				(1)	→ 2

$F = \bar{w}\bar{y}z + x\bar{y}z + \bar{w}\bar{x}y\bar{z} + w\bar{x}yz + wxy\bar{z}$ → Por Diagrama

Por propiedades.

$F = wxy\bar{z} + wx\bar{y}z + w\bar{x}yz + \bar{w}x\bar{y}z + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}\bar{y}z$

$F = wxy\bar{z} + wx\bar{y}z + w\bar{x}yz + \bar{w}\bar{y}z(x+\bar{x}) + \bar{w}\bar{x}y\bar{z}$ ($x+\bar{x}=1$)

$F = wxy\bar{z} + wx\bar{y}z + w\bar{x}yz + \bar{w}\bar{y}z + \bar{w}\bar{x}y\bar{z}$

$F = wxy\bar{z} + z\bar{y}(wx+\bar{w}) + w\bar{x}yz + \bar{w}\bar{x}y\bar{z}$

$F = wxy\bar{z} + z\bar{y}(\bar{w}+x) + w\bar{x}yz + \bar{w}\bar{x}y\bar{z}$

$F = wxy\bar{z} + z\bar{y}\bar{w} + z\bar{y}x + w\bar{x}yz + \bar{w}\bar{x}y\bar{z}$

$F = wxy\bar{z} + \bar{w}\bar{y}z + x\bar{y}z + w\bar{x}yz + \bar{w}\bar{x}y\bar{z}$

(8) Obtener el producto de sumas

i) Resuelto en el video "Simplificación de funcions booleanas".

ii)

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
wx	0		0	
$w\bar{x}$		0	0	0
$\bar{w}\bar{x}$	0		0	
$\bar{w}x$	0	0	0	

$$\overline{F} = \bar{y}\bar{z} + w\bar{x}\bar{y} + w\bar{x}\bar{z} + \bar{w}x\bar{z} + \bar{w}yz + xyz$$

$$\overline{\overline{F}} = \left(\overline{\bar{y}\bar{z} + w\bar{x}\bar{y} + w\bar{x}\bar{z} + \bar{w}x\bar{z} + \bar{w}yz + xyz}\right)$$

Se aplica Complemento miembro a miembro

$$F = \overline{\bar{y}\bar{z}} \cdot \overline{w\bar{x}\bar{y}} \cdot \overline{w\bar{x}\bar{z}} \cdot \overline{\bar{w}x\bar{z}} \cdot \overline{\bar{w}yz} \cdot \overline{xyz}$$

$$F = \left(\bar{\bar{y}} + \bar{\bar{z}}\right) \cdot \left(\bar{w} + \bar{\bar{x}} + \bar{\bar{y}}\right) \cdot \left(\bar{w} + \bar{\bar{x}} + \bar{\bar{z}}\right) \cdot \left(\bar{\bar{w}} + \bar{x} + \bar{\bar{z}}\right) \cdot \left(\bar{\bar{w}} + \bar{y} + \bar{z}\right) \cdot \left(\bar{x} + \bar{y} + \bar{z}\right)$$

$$F = (y+z) \cdot (\bar{w} + x + y) \cdot (\bar{w} + x + z) \cdot (w + \bar{x} + z) \cdot (w + \bar{y} + \bar{z}) \cdot (\bar{x} + \bar{y} + \bar{z})$$

b Producto de Sumas.